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Abstract

The thesis consists of four essays, which make empirical and methodological contributions to the fields of business cycle analysis and demography. The first essay presents insights on U.S. business cycle volatility since 1867 derived from a Bayesian dynamic factor model. The essay finds that volatility increased in the interwar periods, which is reversed after World War II. While evidence can be generated of postwar moderation relative to pre-1914, this evidence is not robust to structural change, implemented by time-varying factor loadings. The second essay scrutinizes Bayesian features in dynamic index models. The essay shows that large-scale datasets can be used in levels throughout the whole analysis, without any pre-assumption on the persistence. Furthermore, the essay shows how to determine the number of factors accurately by computing the Bayes factor. The third essay presents a new way to model age-specific mortality rates. Covariates are incorporated and their dynamics are jointly modeled with the latent variables underlying mortality of all age classes. In contrast to the literature, a similar development of adjacent age groups is assured, allowing for consistent forecasts. The essay demonstrates that time series of covariates contain predictive power for age-specific rates. Furthermore, it is observed that in particular parameter uncertainty is important for long-run forecasts, implicating that ignoring parameter uncertainty might yield misleadingly precise predictions. In the fourth essay the model developed in the third essay is utilized to conduct a structural analysis of macroeconomic fluctuations and age-specific mortality rates. The results reveal that the mortality of young adults, concerning business cycles, noticeably differ from the rest of the population. This implies that differentiating closely between particular age classes, might be important in order to avoid spurious results.

Keywords: Bayesian time series econometrics, dynamic factor models, business cycle measurement, economic history, demography

Abstract

Diese Arbeit besteht aus vier Essays, die empirische und methodische Beiträge zur Messung von Konjunkturzyklen und deren Zusammenhänge zu demographischen Variablen liefern. Der erste Essay analysiert unter Zuhilfenahme eines Bayesianischen Dynamischen Faktormodelles die Volatilität des US-amerikanischen Konjunkturzyklus seit 1867. In dem Essay wird gezeigt, dass die Volatilität in der Periode vor dem Ersten Weltkrieg und nachdem Zweiten Weltkrieg niedriger war als in der Zwischenkriegszeit. Eine geringere Volatilität für die Periode nach dem Zweiten Weltkrieg im Vergleich zu der Periode vor dem Ersten Weltkrieg kann nicht bestätigt werden. Der zweite Essay hebt die Bayesianischen Eigenschaften bezüglich dynamischer Faktormodelle hervor. Der Essay zeigt, dass die ganze Analyse hindurch - im Gegensatz zu klassischen Ansätzen - keine Annahmen an die Persistenz der Zeitreihen getroffen werden muss. Des Weiteren wird veranschaulicht, wie im Bayesianischen Rahmen die Anzahl der Faktoren bestimmt werden kann. Der dritte Essay entwickelt einen neuen Ansatz, um altersspezifische Sterblichkeitsraten zu modellieren. Kovariate werden mit einbezogen und ihre Dynamik wird gemeinsam mit der von latenten Variablen, die allen Alterklassen zugrunde liegen, modelliert. Die Resultate bestätigen, dass makroökonomische Variablen Prognosekraft für die Sterblichkeit beinhalten. Im vierten Essay werden makroökonomischen Zeitreihen zusammen mit altersspezifischen Sterblichkeitsraten einer strukturellen Analyse unterzogen. Es wird gezeigt, dass sich die Sterblichkeit von jungen Erwachsenen in Abhängigkeit von Konjunkturzyklen deutlich von den der anderen Alterklassen unterscheidet. Daher sollte in solchen Analysen, um Scheinkorrelation vorzubeugen, zwischen den einzelnen Altersklassen differenziert werden.

Keywords: Bayesianische Zeitreihenanalyse, dynamische Faktormodelle, Datierung von Konjunkturzyklen, Wirtschaftsgeschichte, Demographie

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1 Introduction

Measuring economic activity is a formidable task. The same is true for predicting and analyzing mortality with the help of economic variables. Fortunately, both tasks can be accomplished with Bayesian methods. This thesis employs Bayesian state space models and demonstrates in four essays, how these models can be utilized to provide possible alternative measures for (Historical) National Accounts, predict age-specific mortality rates, and investigate the interaction of aggregate economic variables with mortality.

The first essay studies the volatility of the U.S. business cycle for the period 1867–1995 using a Bayesian dynamic factor (index) model with time-varying parameters. The essay concludes that for long run business cycles analysis time-varying parameters are crucial. In contrast to existing Historical National Accounts estimates and to a constant parameter framework, the model with time-varying parameters avoids the spurious correlation problem raised by Romer [1986, 1988]. This turns out to be important when comparing pre World War I with post World War II business cycle volatility.

The second essay contributes methodologically to the existing literature of dynamic index models. Opposed to the classical time series econometrics, Bayesians do not distinguish between stationary and nonstationary data. This feature is exploited and deployed to the estimation of dynamic index models. It demonstrates that large-scale datasets can be analyzed without paying attention to the stationarity of the data, deriving the uncertainty around the parameters and the unobserved indices of the model in a natural way. Furthermore, it shows how to determine the number of factors using the Bayes factor.

The third essay focuses on modeling and forecasting age-specific mortality rates. Therefore it extends the well known Lee and Carter [1992] approach in two respects. First, by including macroeconomic time series, which leads to an improved forecast performance. Second, by smoothing the parameters of the age-dimension, which avoids that the predictions of adjacent age-classes diverge from each other. Furthermore, it is demonstrated that the uncertainty concerning long run forecasts is misleading, when parameter uncertainty is not considered.

Essay four applies the method developed in the third essay and provides a short run structural analysis of age-specific mortality rates and macroeconomic time series for post World War II U.S. data. It can be observed that the business cycle affects the mortality of young adults differently than the mortality of all other age classes. To avoid spurious regression, the results implicate that it should be differentiated between particular age classes, when studying the influence of the business cycle on mortality.

From a technical point of view all essays rely on Markov Chain Monte Carlo (MCMC) methods, especially the Gibbs sampling algorithm. The Gibbs sampling algorithm (Geman and Geman, 1984, Gelfand and Smith, 1990) is a procedure which enables the researcher to draw a random variable from a distribution indirectly, without the need to calculate the density. The Gibbs sampling procedure hence allows to study the high-dimensional models described in this thesis, making inference about model parameters or predictions feasible. Furthermore, as all models include unobserved variables, they can be accommodated into a state space set up. To extract the unobserved components the state space system is estimated using the Gaussian Kalman filter and smoother. In the following, an introductory review of each of the essays is provided.

1.1 Review of Chapter 2

Chapter 2 is joint work with Albrecht Ritschl and Martin Uebele. An earlier version has appeared as No. 7069 in the *CEPR Discussion Paper Series.* Chapter 2 studies the U.S. business cycle over the period 1867–1995. It contributes to the debate, concerning the comparison of pre World War I and post Word War II business cycle volatility. The debate centers around two different Historical National Accounts (HNA) series and their implications for U.S. business cycle volatility since the 19th century. The first series, which is constructed by Balke and Gordon [1986, 1989], exhibits high volatility before World War I, compared to the rather moderate fluctuations of postwar GNP. The alternative HNA series, which is constructed by Romer (1986, 1988), challenges this view based on a revision of the alternative series of Kendrick [1961], implying that there was no postwar moderation relative to the pre-World War I years.

Chapter 2 offers an alternative but complementary approach to measuring the volatility of the U.S. business cycle in the very long run. It employs a time-varying Bayesian dynamic factor (index) model to extract a measure of economic activity from a broad database, aggregating a large amount of disaggregate information. Disaggregate series are often abundant for historical periods, but usually do not match national accounting categories well, and the information needed for proper aggregation is incomplete. As a consequence, proxies have to be used, which can be controversial as mentioned above. The dynamic factor approach replaces the questionable aggregation techniques used in the construction of HNAs with a statistical aggregator. Series that would be of limited use in reconstructing HNAs can now be exploited for their business cycle indicator characteristics.

The evolution of U.S. business cycle volatility over time is studied in two exercises. The first exercise covers the full sample from 1867 to 1995. In the second exercise, the change in volatility across World War I to 1929 is examined. Results are compared to the HNA reconstructions of GDP for the pre-1929 era by Balke and Gordon [1989] and Romer [1989]. In the first exercise, 53 time series that are constructed on an unchanged methodological basis are included. For the second exercise, a wider panel of 98 such series is utilized. Data are taken from the Historical Statistics of the U.S., see Carter et al. [2006], as well as the NBER's Macrohistory Database, dating back to the business cycle project of Burns and Mitchell [1946].

The findings in Chapter 2 suggest no overall postwar moderation relative to the pre-World War I period. The results are confirmed by the study of sectoral indices, except for agriculture and services. This is informative about existing HNA estimates, where the proper way to include these two sectors was disputed. Nominal factors are also specified, revealing postwar moderation in the nominal series compared to pre-1914. At the same time, the 1970s were more volatile than the period of the classical Gold Standard before World War I. The standard evidence on reduced volatility after the 1980s (e.g., Cogley and Sargent, 2005; Primiceri, 2005) is replicated.

1.2 Review of Chapter 3

Chapter 3 studies dynamic index models, which decompose a given series into a common and idiosyncratic component, from a Bayesian perspective. The fact that the stationarity of a series is of no importance in Bayesian econometrics is exploited. The case of a unit root is treated as just one of many possibilities: conditional on the data, the unit root case merely obtains a certain posterior weight. The model is estimated with the Gibbs sample algorithm in an efficient one-step estimation procedure. Furthermore, the appropriate number of factors is determined, relying on the comparison of predictions of the competing model specifications (e.g., Jeffreys, 1961). The Bayes factor is calculated for the index model, using the procedure described in Chib [1995].

The method described in Chapter 3 can be seen as a Bayesian alternative to the likelihood-based approach of Quah and Sargent [1993] and the principal component approaches suggested by Forni and Reichlin [1998] and Bai [2004]. Working with Bayesian methods allows a straightforward derivation of the uncertainty around the factors and parameters. In particular, the parameter uncertainty might be important when it comes to forecasting economic series, as it leads to incorrectly precise predictions (e.g., Uhlig, 1994; Chapter 4).

To test the accuracy of the approach the model is applied to an artificial dataset. The dataset consists of a common stochastic trend with drift and a common transitory component. Even though no restrictions are imposed on the dynamics of the model, the method is able to distingiush between the common stochastic trend with drift and the common transitory component. Both, the common stationary and the nonstationary component, are estimated very precisely. Furthermore, using the Bayes factor approach, the number of factors are estimated accurately as well. In a second step, the Bayesian dynamic index model is applied to post WWII U.S. data. The dataset has recently been compiled by Stock and Watson [2008] and consists of 108 macroeconomic time series. Opposed to Stock and Watson [2008], the data are analyzed in levels and are not transformed to eliminate trends. The Bayes factor for the Stock-Watson dataset is calculated to determine the number of factors. The procedure recommends to choose seven factors. Visual inspection reveals that the procedure is able to estimate the factors very precisely and that it automatically decomposes the data into common trend and cyclical components. Thus, the method discussed in Chapter 3 allows to use datasets with large cross sections already in levels, without worrying about the presence of a unit root.

1.3 Review of Chapter 4

The papers underlying Chapter 4 and Chapter 5 are joint work with Wolfgang Reichmuth. Chapter 4 provides a novel approach of modeling and forecasting age-specific mortality rates. It extends the popular Lee-Carter model with covariates and tackles several difficulties, which emerge in the existing literature, concerning the age dimension. The model used in Chapter 4 nests the widely applied Lee-Carter approach and statistical models, which incorporate covariates. A Bayesian estimation procedure is applied, using a simulation-based approach. To link the unobserved common component of the age-specific mortality rates and the covariates vector autoregressions (VAR) are employed, capturing their dynamic interactions. In addition, the age-dimension of the demographic variables are smoothed with the help of AR-processes, which results into nondiverging predictions of age-specific mortality rates. This has not been done so far in the literature.

The model is applied to age-specific U.S. mortality, incorporating U.S. GDP and unemployment as covariates. In-sample forecasts are used to test the performance of the model, confirming that the model works accurately. In an out-of-sample forecasting exercise it is shown that, concerning mortality rates, the macroeconomic variables have predictive power. Moreover, due to the assumed law of motion, smooth transitions for the age dimension can be observed. Employing Bayesian methods, it is straightforward to decompose the forecast results into parameter and residual uncertainty. It can be observed that uncertainty regarding the residuals is important in the short-run, whereas parameter uncertainty dominates long-run forecasts. This implicates that ignoring parameter uncertainty might result into misleadingly precise predictions especially in the long run.

The field of application of the model described in Chapter 4 is not restricted to reduced form forecasts only. It also allows for a structural analysis. For instance, the reactions of age-specific demographic variables to particular macroeconomic shocks are deduced. This is the subject of Chapter 5.

1.4 Review of Chapter 5

In Chapter 5 the model desribed in Chapter 4 is used to conduct a structural analysis of mortality rates and macroeconomic variables. A structural vector autoregression is employed to shed light on the effects of economic shocks on mortality rates for all age classes, which is neglected in the literature so far. Moreover, instead of using unemployment as the only business cycle variable - as it is usually done - GDP growth is added as a further covariate. The consequences of business cycle movements on age-specific mortality are studied, using anual data for the United States since 1956. In addition, data since 1933 covering the long aftermath of the Great Depression and World War II are included. Datasets for France and Japan allow to draw an international comparison of the particular relations between mortality and the business cycle in countries with different economic and demographic experiences and very different institutional settings.

The model allows to analyse the relationship between the common unobserved mortality variable and the different covariates by means of impulse response functions. Based on these interactions and the corresponding coefficients, which link the variables to age-specific mortality, reactions of all age classes to shocks in the economic variables are calculated. The reactions for a fixed age as well as of a cohort, aging over time, are computed. In all cases, the Bayesian estimation approach yields not only point estimates, but information on the whole distribution of the results. Thus, error bands with probability masses corresponding to different percentiles of the responses of age-specific mortality are presented.

Recently, a debate on the effects of the business cycle on mortality was kicked off by recent findings of procyclical mortality, contradicting conventional socio-epidemiological wisdom. Chapter 5 contributes to this debate. It finds that for the United States in the period 1956–2004 the reaction of the male 20 to 30 years olds to macroeconomic shocks constitutes an exception. While most other age classes react negatively to a shock in unemployment and positively to a shock in GDP growth, the 25 years olds react with reversed signs. These findings are confirmed with international data from France and Japan, observing that in both countries - in addition to the young male adults - even the 30 to 40 years olds react differently. This suggests that when examining the relationship between business cycles and mortality, data should include all single age classes in order to avoid spurious results. To investigate a change in the relationship between macroeconomic variables and mortality rates data for the period 1933–1969 are used. The calculated impulse response functions reveal that the relationship has indeed evolved over time. Unlike the 1956–2004 sample, all mortality age classes react procyclically in the short-term and for the most part countercyclically in the mid-term.

2 The U.S. Business Cycle, 1867-1995: A Dynamic Factor Approach

This chapter presents insights on U.S. business cycle volatility since 1867 derived from diffusion indices. We employ a Bayesian dynamic factor model to obtain aggregate and sectoral economic activity indices. We find a remarkable increase in volatility across World War I, which is reversed after World War II. While we can generate evidence of postwar moderation relative to pre-1914, this evidence is not robust to structural change, implemented by time-varying factor loadings. We do find evidence of moderation in the nominal series, however, and reproduce the standard result of moderation since the 1980s. Our estimates broadly confirm the NBER historical business cycle chronology as well the National Income and Product Accounts, except for World War II where they support alternative estimates of Kuznets (1952).

2.1 Introduction

Measuring the American business cycle in the long run has been the subject matter of much debate. While there is broad agreement on the business cycle turning points, the issue of volatility is still not fully resolved, as different available estimates yield contradictory results. How severe were the key recessions other than the Great Depression of the 1930s, that is, the recessions of the mid 1880s, of 1907, and of 1920/21? Was wartime prosperity in the mid-1940s really so strong? And has the U.S. business cycle become more moderate since World War II, not just with respect to the interwar period but also compared to the prewar years?

Researchers have disagreed on the severity of the downturn after World War I as well as on the other two questions. Following Burns [1960], DeLong [1984] argued that business fluctuations after World War II were more moderate than before World War I, and certainly during the interwar period. This view was challenged in a series of papers by Romer [1986, 1988], who argued that postwar stabilization relative to the decades before World War I was an artifact of the historical output and unemployment data.

Given the lack of reliable aggregate series for the decades before 1929 when the official National Income and Product Accounts (NIPA) set in, existing evidence was based on Historical National Account (HNA) estimates. Most of the debate evolved around two rivaling such series and their implications for U.S. business cycle volatility since the 19th century. Balke and Gordon [1986, 1989] modified a popular GNP series originating from the Commerce Department, for which they produced a widely used quarterly interpolation. The high volatility of this series before World War I, compared to the rather moderate fluctuations of postwar GNP, is what shaped conventional wisdom in the 1980s. Romer (1986, 1988) challenged this view based on a revision of the alternative series of Kendrick [1961], which she argued was less prone to spurious volatility.[1] Her results implied that there was no postwar moderation relative to the pre-World War I years. However, her own calculations have been criticized for depending on assumptions which are not empirically testable given the lack of historical GNP data, see Lebergott [1986]. Following Kim and Nelson [1999b], McConnell and Perez-Quiros [2000], Blanchard and Simon [2001], and Stock and Watson [2002], research on the stabilization of the U.S. business cycle has therefore focused mostly on moderation within the postwar period itself.

The present chapter offers an alternative but complementary approach to measuring the volatility of the U.S. business cycle in the very long run. We draw on the growing literature on diffusion indices (using a term of Stock and Watson [1998]) of economic activity, which are distilled from a large panel of disaggregate time series using dynamic factor analysis (DFA). Stock and Watson [1991] developed an unobserved component model for disaggregate series representing the U.S. postwar economy which reliably replicates the NBER's business cycle turning points.[2] Factor models have become popular as an alternative to national accounts because they aggregate a large amount of disaggregate information and are less affected by data revisions than national accounts.[3] The same issues loom large with historical data. Disaggregate series are often abundant for historical periods, but usually do not match national accounting categories well, and the information needed for proper aggregation is incomplete. As a consequence, proxies have to be used, which can be controversial as mentioned above. The DFA approach

[1] Both the Commerce and the Kendrick series are related to earlier work by Kuznets [1941, 1946], see Romer [1988] for a discussion.

[2] Stock and Watson [1998] analyzed 170 series successfully forecasting U.S. postwar CPI and IP.

[3] Romer [1991] estimated a factor model with principal components, however on a narrower and shorter data base. Her findings are comparable to ours.

replaces the questionable aggregation techniques used in the construction of HNAs with a statistical aggregator. Series that would be of limited use in reconstructing HNAs can now be exploited for their business cycle indicator characteristics, i.e. their contribution to the common component. To our knowledge, this approach was first applied in the context of presenting an alternative to HNA estimates by Gerlach and Gerlach-Kristen [2005] for Switzerland between the 1880s and the Great Depression of the 1930s. Sarferaz and Uebele [2007] employ a Bayesian dynamic factor model to obtain an index of economic activity for 19th century Germany, comparing it to different rivaling HNA-based chronologies. The present chapter extends this methodology to the historical application of macroeconomic diffusion indices with time-varying factor loadings. This helps to capture structural change, which is important if long time spans are to be covered.

In this chapter, we study the evolution of U.S. business cycle volatility over time in two exercises. The first exercise covers the full sample from 1867 to 1995. In the second exercise, we examine the change in volatility across World War I to 1929. Results are compared to the HNA reconstructions of GDP for the pre-1929 era by Balke and Gordon [1989] and Romer [1989]. In the first exercise, we include 53 time series that are constructed on an unchanged methodological basis. For the second exercise, we employ a wider panel of 98 such series. Data are taken from the Historical Statistics of the U.S., see Carter et al. [2006], as well as the NBER's Macrohistory Database, which itself dates back to the business cycle project of Burns and Mitchell [1946].

Our findings suggest no overall postwar moderation relative to the pre-World War I period. We introduce identifying restrictions to study sectoral indices separately and find our results confirmed, except for agriculture and services. This is informative about existing HNA estimates, where the proper way to include these two sectors was disputed. We also specify nominal factors and find evidence in favor of postwar moderation in the nominal series compared to pre-1914. At the same time, the 1970s were more volatile than the period of the classical Gold Standard before World War I. We replicate the standard evidence on reduced volatility after the 1980s (see e.g. Cogley and Sargent [2005] Primiceri [2005]). We also obtain new results on the 1921 slump, as well as the wartime boom during World War II.

The remainder of the chapter is structured as follows. The next section briefly sketches the Bayesian factor model. Section 2.3, divided up in several subsections, presents the evidence. Section 2.4 concludes. Data and technical details are discussed in the appendix.

2.2 A Bayesian Dynamic Factor Model

2.2.1 The Model

Dynamic factor models in the vein of Sargent and Sims [1977], Geweke [1977] and Stock and Watson [1989] assume that a panel dataset can be characterized by a latent common component that captures the comovements of the cross section, and a variable-specific idiosyncratic component. These models imply that economic activity is driven by a small number of latent driving forces, which can be revealed by estimation of the dynamic factors. A Bayesian approach to dynamic factor analysis is provided by Otrok and Whiteman [1998] and Kim and Nelson [1999a], amongst others.
Following Del Negro and Otrok [2003] our panel of data Y_t , spanning a cross section of N series and an observation period of length T, is described by the following observation equation:[4]

$$Y_t = \Lambda_t f_t + U_t \tag{2.1}$$

where f_t represents a 1×1 latent factor, while Λ_t is a $N \times 1$ coefficient vector linking the common factor to the i-th variable at time t, and U_t is an $N \times 1$ vector of variable-specific idiosyncratic components. The latent factor captures the common dynamics of the dataset and is our primary object of interest.[5]. We assume that the factor evolves according to an AR(q) process:

$$f_t = \varphi_1 f_{t-1} + \ldots + \varphi_q f_{t-q} + \nu_t \tag{2.2}$$

with $\nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$. The idiosyncratic components U_t are assumed to follow an AR(p) process:

$$U_t = \Theta_1 U_{t-1} + \ldots + \Theta_p U_{t-p} + \chi_t \tag{2.3}$$

[4]Del Negro and Otrok [2003] estimate the parameters of the model unconditional on the initial observations, using a Metropolis-within-Gibbs sampler. We estimate the parameters of the model conditional on the initial observations, using the Gibbs sampling approach.
[5]Generalization to several factors is straightforward.

where $\Theta_1, \ldots, \Theta_p$ are $N \times N$ diagonal matrices and $\chi_t \sim \mathcal{N}(0_{N\times 1}, \Omega_\chi)$ with

$$\Omega_\chi = \begin{bmatrix} \sigma^2_{1,\chi} & 0 & \cdots & 0 \\ 0 & \sigma^2_{2,\chi} & \vdots & \vdots \\ \vdots & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma^2_{N,\chi} \end{bmatrix}$$

The factor loadings or coefficients on the factor in equation (2.1), Λ_t, are assumed to be either constant or (in the time-varying model) follow a driftless random walk:

$$\Lambda_t = \mathcal{I}_N \Lambda_{t-1} + \epsilon_t \tag{2.4}$$

where $\mathcal{I}_N$ is a $N \times N$ identity matrix and $\epsilon_t \sim \mathcal{N}(0_{N\times 1}, \Omega_\epsilon)$ with

$$\Omega_\epsilon = \begin{bmatrix} \sigma^2_{1,\epsilon} & 0 & \cdots & 0 \\ 0 & \sigma^2_{2,\epsilon} & \vdots & \vdots \\ \vdots & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma^2_{N,\epsilon} \end{bmatrix}$$

and where the disturbances χ_t and ϵ_t are independent of each other.

The dynamic factor in this model is identified up to a scaling constant and a sign restriction. We deal with scale indeterminacy by normalizing the standard deviation of the factor innovations to $\sigma_\nu = 1$. The sign indeterminacy of the factor loadings Λ_t and the factor f_t is resolved by a sign convention, i.e. by restricting one of the factor loadings to be positive (see Geweke and Zhou [1996]). Neither operation involves loss in generality.

2.2.2 Priors

Before proceeding to the estimation of the system, we specify prior assumptions. These priors are informative and have a substantive interpretation in terms of our research question, especially with regard to time variation in the parameters. We adopt priors for four groups of parameters of the above system. These are, in turn, the parameters in the factor equation (2.2), the parameters in equation (2.3) governing the law of motion of the idiosyncratic component, the parameters in the law of motion of the factor loadings (2.4) and the parameters in the observation equation (2.1).

For the AR parameters $\varphi_1, \varphi_2, \ldots, \varphi_q$ of the factor equation, we specify the following prior:

$$\varphi^{prior} \sim \mathcal{N}(\underline{\varphi}, \underline{V}_{\varphi})$$

where $\underline{\varphi} = 0_{q\times 1}$ and

$$\underline{V}_{\varphi} = \tau_1 \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \frac{1}{2} & \vdots & \vdots \\ \vdots & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{q} \end{bmatrix}$$

Analogously, for the AR parameters $\Theta_1, \Theta_2, \ldots, \Theta_p$ of the law of motion of the idiosyncratic components, we specify the following prior:

$$\theta^{prior} \sim \mathcal{N}(\underline{\theta}, \underline{V}_{\theta})$$

where $\underline{\theta} = 0_{p\times 1}$ and

$$\underline{V}_{\theta} = \tau_2 \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \frac{1}{2} & \vdots & \vdots \\ \vdots & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{p} \end{bmatrix}$$

We choose $\tau_1 = 0.2$ and $\tau_2 = 1$. Both priors imply that we punish more distant lags on the autoregressive terms, very much in the spirit of the Litterman prior, see Doan et al. [1984]. This is implemented by progressively decreasing the uncertainty about the mean prior belief that the parameters are zero as lag length increases.

For the variances of the disturbances in χ_t, we specified the following prior:

$$\sigma_{\chi}^{2\ prior} \sim \mathcal{IG}\left(\frac{\alpha_{\chi}}{2}, \frac{\delta_{\chi}}{2}\right)$$

We choose $\alpha_{\chi} = 6$ and $\delta_{\chi} = 0.001$, which implies a fairly loose prior. $\mathcal{IG}$ denotes the inverted gamma distribution.

For the factor loadings, we distinguish two cases. With constant factor loadings (disregarding structural change), the relevant prior for each individual factor loading is:

$$\lambda^{prior} \sim \mathcal{N}(\underline{\lambda}, \underline{V}_{\lambda})$$

where $\underline{\lambda} = 0$ and $\underline{V}_\lambda = 100$.

With time-varying factor loadings, for each of the variances of the disturbances in ϵ_t the prior is:

$$\sigma_\epsilon^{2\ prior} \sim \mathcal{IG}\left(\frac{\alpha_\epsilon}{2}, \frac{\delta_\epsilon}{2}\right)$$

We chose α_ϵ and δ_ϵ so as to capture longer term structural variation by changing factor loadings, while volatility at the relevant business cycle frequencies is assigned to movements in the factors.[6]

2.2.3 Estimation

We estimate the model in Bayesian fashion via the Gibbs sampling approach. This procedure enables the researcher to draw from nonstandard distributions by splitting them up into several blocks of standard conditional distributions. In our case, the estimation procedure is subdivided into three blocks: First, the parameters of the model c, φ, θ_r for $s = 1, \ldots, q$ and $r = 1, \ldots, p$ are calculated. Second, conditional on the estimated values of the first block, the factor f_t is computed. Finally, conditional on the results of the previous blocks we estimate the factor loadings. After the estimation of the third block, we start the next iteration step again at the first block by conditioning on the last iteration step. [7] These iterations have the Markov property: as the number of steps increases, the conditional posterior distributions of the parameters and the factor converge to their marginal posterior distributions at an exponential rate (see Geman and Geman [1984]).

2.3 Empirical Results

Estimates were obtained for lag lengths $p = 1, q = 8$, taking 30,000 draws and discarding the first 9,000 as burn-in. Specifications with constant and time-varying factor loadings are reported alongside each other. Convergence of the Gibbs sampler was checked by varying the starting values and comparing the results. All series were detrended using the Hodrick-Prescott filter with the (6.25) parameters suggested by Ravn and Uhlig [2002] for business cycle frequencies, and were subsequently standardized.[8]

[6] We work with $\alpha_\epsilon = 100$ and $\delta_\epsilon = 1$, which generated a good fit for the postwar data.
[7] See the appendix for a more detailed description of the estimation procedure.
[8] We also tried Christiano and Fitzgerald [2003] and Baxter and King [1999] filters as well as differencing, with little change in results. Data sources are listed in Appendix 7.4.

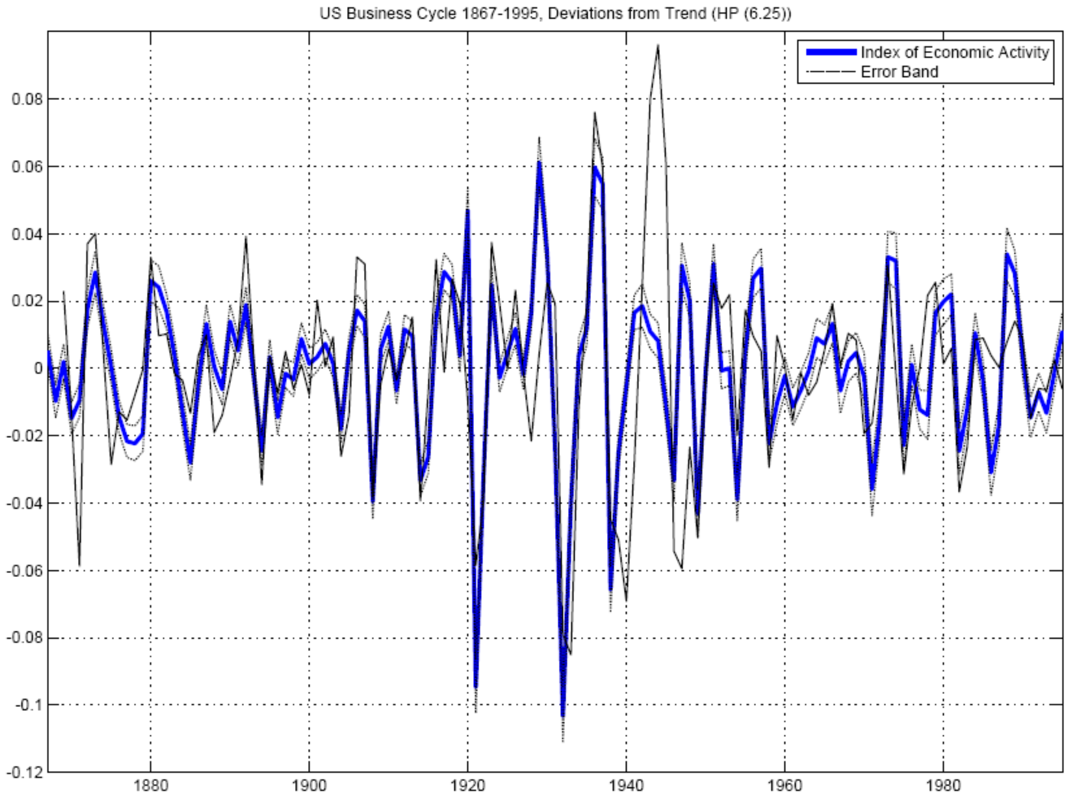

Figure 2.1: The U.S. business cycle, 1867-1995. Factor vs GNP (1869-1929 Romer (1989), 1930-1995 NIPA). Factor from 53 series. All data are deviations from HP(6.25) trend, FL-prior: 1/100; i.e. tight around 1%.

2.3.1 The U.S. Business Cycle in the Long Run

Figure 2.1 is our representation of the American business cycle between 1867 and 1995. It shows a one-factor model of aggregate economic activity, obtained from 53 consistent time series available for that period. The official NIPA series of GDP starting in 1929 and a GDP estimate of Romer (1989) for 1867-1929 are shown for comparison. The factor is calibrated to the standard deviation of NIPA from its HP (6.25) trend for 1946-1995.

As the Figure shows, the factor captures the business cycle turning points in GDP quite well. This is true for both the postwar period and the historical business cycles and the 19th century (see Miron and Romer [1990], Davis [2004] and Davis et al. [2004] for details on the chronology.)

Differences with the GDP data emerge around the World Wars. The recession of 1920/21 comes out more strongly than in the GDP estimates of Romer [1988] and Balke and Gordon [1989]. Also, our factor does not show the peak in the NIPA estimate of GDP during World War II. We will discuss these results in more detail below.

The factor shown in Figure 2.1 is based on conservative assumptions about the degree of time variation in the factor loadings. As we are interested in historical volatility comparisons, our approach is to restrict time variation in factors loadings to low-frequency structural changes, such that volatility at the relevant business cycle frequencies is captured by the factors themselves. Figure 2.2 shows the factor loadings for our 53 series under our preferred conservative prior against a more diffuse alternative. As can be seen, the tight prior allows for smooth changes in the factor loadings while suppressing volatility at business cycle frequencies. In contrast, cyclical components are present in the factor loadings under the loose prior, which would affect the volatility of the factor at the relevant frequencies and is therefore discarded.

The factor in Figure 2.1, representing aggregate activity, is our yardstick for intertemporal comparisons of U.S. business cycle volatility. Table 2.1 compares volatility in the post-World War II period to the pre-World War I era. Results are provided for both constant and time-varying factor loadings. The GDP estimates of Romer [1989] and Balke and Gordon [1986, 1989] , designed to extend the NIPA data on GDP backwards from 1929, provide the relevant comparison for the period prior to World War I.

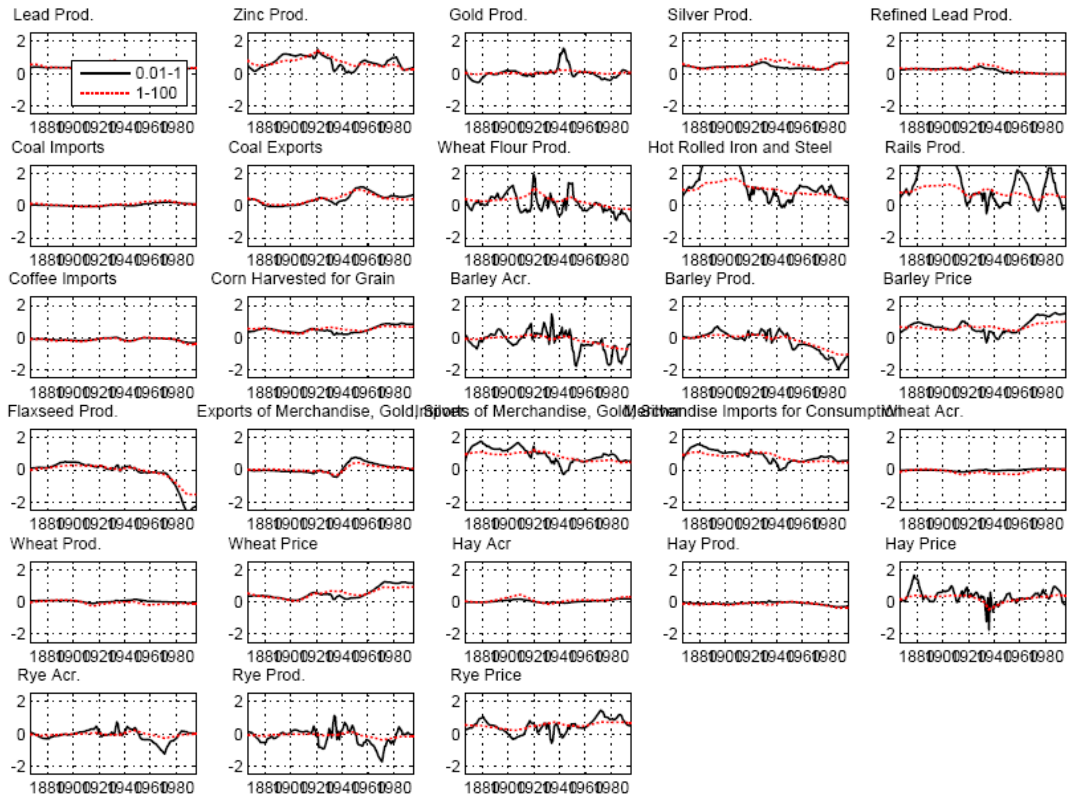

Figure 2.2: Factor Loadings, 1867-1995. Tight prior (red dotted line): $\delta_\epsilon = 1, \alpha_\epsilon = 100$. Loose prior (black continuous line): $\delta_\epsilon = 0.01, \alpha_\epsilon = 1$. Both priors imply the same mean of the IG distribution.

Table 2.1: Volatility Comparison, Post-World War II / Pre-World War I. Factor vs. GDP Estimates

Dev. from HP(6.25)-trend %	1867 -1913	1946 -95	Post-WW II /Pre-WW I
Romer GDP / NIPA	2.07	2.01	0.97
Balke/Gordon GDP / NIPA	2.47	2.01	0.81
FACTOR, ALL 53 SERIES			
Constant	2.00	2.01	1.01
Time Varying	1.51	2.01	1.33
FACTOR, NON-AGRICULTURAL REAL SERIES			
Constant	2.20	1.87	0.85
Time Varying	1.24	1.87	1.52
FACTOR, AGRICULTURAL REAL SERIES			
Constant	3.21	6.87	2.14
Time Varying	9.37	6.87	0.74
FACTOR, REAL NON-PHYSICAL OUTPUT SERIES			
Constant	1.46	2.01	1.38
Time Varying	1.84	2.01	1.09
FACTOR, NOMINAL SERIES			
Constant	1.32	1.62	1.23
Time Varying	1.93	1.62	0.84
FACTOR, NONAGR NOMINAL SERIES			
Constant	1.84	1.17	0.64
Time Varying	1.34	1.17	0.87
FACTOR, NONAGR NOMINAL SERIES			
Constant	7.17	8.30	1.16
Time Varying	7.53	8.30	1.10

In Table 2.1, the volatility of all data is calibrated to NIPA for the postwar period. For the prewar period, Balke/Gordon's GDP estimate is more volatile than postwar GNP, indicating postwar moderation in the U.S. business cycle. Romer's (1989) estimate of pre-1914 GDP is less volatile, which suggests no postwar moderation relative to the prewar business cycle.

Table 2.1 reports two versions of our factor model, one with constant, the other with time varying factor loadings. For constant factor loadings, the factor indicates no change in postwar volatility relative to the prewar period. In this, it reproduces Romer's (1989) results. For time-varying factor loadings, the prewar business cycle becomes even less volatile than in Romer's estimate. This would imply that the U.S. postwar business cycle was probably more, not less volatile than before World War I.

Yet we can also reproduce Balke/Gordon's (1986, 1989) postwar moderation result. To this end, we focus on a subset of the data that is closest to their GDP estimate. Under constant factor loadings, a factor for non-agricultural real series (see Table 2.1) exhibits substantial postwar moderation in volatility, close to the reduction implied by the Balke and Gordon [1986, 1989] data. Indeed, their estimate (and the Commerce series of GDP on which it is based) relies heavily on industrial output, as pointed out by Romer [1986, 1989]. The comovement of these series, assuming constant weights, generates moderation across the World Wars also in our factor model. However, this result is not robust to allowing time variation in weights. Under time varying factor loadings as shown in Table 2.1, postwar volatility is again higher than before World War I.

While in both cases, postwar volatility comes out higher relative to pre-1914 if time-varying factor loadings are assumed, this is not always the case. A counterexample is provided by agricultural production. Under constant factor loadings, a factor model of agriculture shows a strong increase in volatility across the World Wars. Time varying factor loadings yield the opposite result, making the postwar agricultural cycle seem strongly muted relative to the pre-World War I period (see Table 2.1). We find this to be reassuring, as increasing agricultural productivity would allow farmers to shift away from the cultivation of weather-dependent and disease-prone crops, thus helping to reduce the volatility of agricultural output. Such a shift would imply changes in the composition of output, which are better captured by time-varying factor loadings.

We obtain a similar effect for the transport and communication series in our dataset. Constant factor loadings would suggest an almost 40% increase in volatility of a suitably identified factor across the World Wars. Including such series in a physical product estimate of pre-war GDP, as suggested by Romer [1989], will therefore tend to lower or eliminate the postwar moderation that is implicit in the indiustrial output series. The lower volatility of Romer's own, broader GDP estimate relative to the physical-output estimate underlying the Balke and Gordon [1986] series is thus reflected in our sectoral results. However, once we allow the factor loadings to vary over time, the volatility increase in these non-production series almost disappears.

The above sectoral factors contribute to an explanation of why the Balke/Gordon and Romer estimates of pre-war GDP differ in volatility. While the former relies more strongly on industrial output, the latter gives higher weight to agriculture and services. Given the low pre-war volatility of the two latter sectors, a broader aggregate obtained under constant weights will necessarily reduce or close the volatility gap that exists in the Balke/Gordon series.

However, introducing time varying factor weights shows that the sectoral discrepancies between pre- and postwar volatility are not the only effect, and not even the dominant one. What matters more is the near-inevitable assumption of constant weights in existing Historical National Accounts for the U.S. Romer [1988, 1989] attempted to overcome this constraint by backward-extrapolating postwar trends in weighing schemes to the pre-World War I estimates. We obtain similar and more pronounced results by allowing slow time variation in the factor loadings, which constitute the weighing scheme of the factor model. As soon as time variation is introduced, a statistical aggregator of economic activity suggests less volatile business cycles in the 19th century than existing estimates, and hence no moderation in the U.S. business cycle across the World Wars.

Similar index problems are present in the long run volatility comparison of the nominal series. A factor obtained from these series under constant factor loadings is essentially a Laspeyres price index. As Table 2.1 bears out, this index would indicate increased nominal volatility in the postwar period. This would be in line with Balke and Gordon [1989], who presented a novel GNP deflator which was substantially less volatile before World War I than previous deflators, thus challenging an older conventional wisdom about high price volatility under the Gold Standard.

However, this finding is again not robust to introducing time variation in the factor loadings. If, as before, we allow for a moderate degree of time variation on the factor loadings, there is postwar moderation relative to pre-1914 in the nominal series. This would lend renewed support to traditional views of price level volatility under the Gold Standard.

Drawing the results of this section together, our principal findings appear to depend on whether or not we account for structural change. If we assume time-invariant factor loadings, our results suggest postwar moderation in real economic activity but not in the nominal series. This would underscore the results of Balke and Gordon [1989], in spite of using a rather different technique. However, as soon as time variation in the factor loadings is permitted, we obtain the opposite result of postwar moderation in the nominal series, but not in overall economic activity. This appears to be consistent with claims of Romer [1989], who argued for the need to account for changing weighting patterns. Our own approach toward time-varying index weights is quite different from hers but seems to confirm her principal conclusions.

2.3.2 The U.S. Business Cycle Across World War I

As a robustness check for the above results, this section focuses on changes in business cycle volatility across World War I. Comparing the pre-1914 years with the interwar period has several advantages. First, it allows us to use a substantially larger dataset of 98 series covering the period from 1867 to 1939 on a consistent basis. Second, choosing the interwar years as the reference period also eliminates possible bias in representing postwar volatility. The GNP data in Balke and Gordon [1986, 1989] bear out a substantial increase in volatility across World War I, while the estimates by Romer [1988] suggested the increase was much weaker. The discrepancy between their findings is partly related to the recession of 1920/21, which is rather mild in Romer's data. In contrast, Balke and Gordon [1989] report a more severe slump.

In the following, we repeat the above exercise for the subperiods from 1867 to 1929 and 1867 to 1939. For the pre- and interwar period, we have a wider dataset of 98 series at hand. To maintain comparability, we will also reestimate the factor model with the narrower dataset of 53 series employed in the previous section. As the results of the previous section were shown to depend so much on time variation in the aggregation procedure, we will again examine constant and time varying loadings alongside each other. The volatility of both factors is calibrated to that of the Balke and Gordon series, obtained as the standard deviation of the cyclical component from a HP(6,25) filter. Figure 2.3 shows the cyclical components in both series alongside the factors (blue

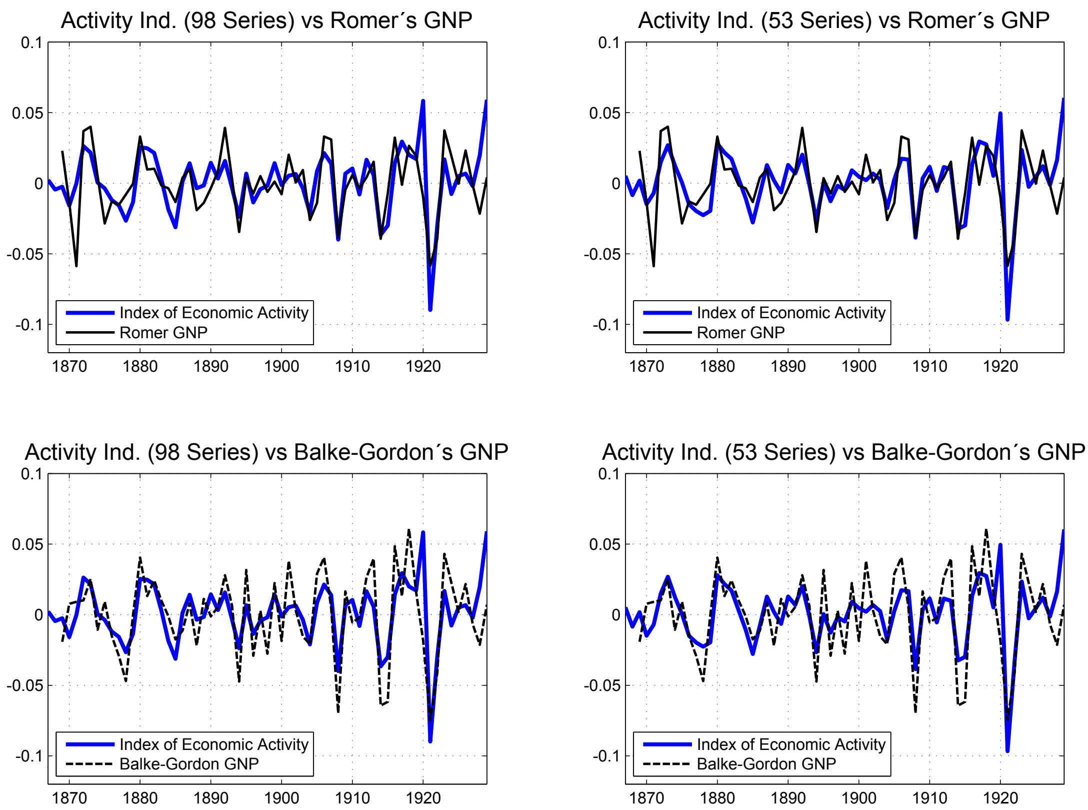

Figure 2.3: The U.S. business cycle 1867-1929, Factor (“activity index”) vs GNP estimates. Factor from 53 and 98 series, respectively. All date are deviations from HP(6.25) trend, FL-prior: 1/100; i.e. tight around 1%.

lines) from 1867-1929 . Comparisons with Romer’s (1989) real GNP measure are shown in the upper panel, while the lower does the same with the Balke and Gordon [1989] GNP estimate.

Two things stand out from this comparison. For the pre-1913 period, the Romer estimate of GDP seems to be more in line with our factor estimates than the Balke and Gordon estimate. For the period from 1914 to 1929, our factors are closer to the Balke and Gordon series than to the Romer estimate. This is particularly true for the slump of 1921, which according to the Balke and Gordon data pushed the cyclical component of output down by almost 9%, compared to only 5% in the Romer [1989] estimate. We also note that the factor indicates a major upturn in the second half of the 1920s, an effect that is missing from both of the rivaling GDP estimates. This evidence would, however, be consistent with a reconstructed index of indsutrial production by Miron and Romer [1990].

Table 2.2: Volatility Comparison Across World War I (1867-1929).

Std.Dev. from HP(6.25) Trend	1867 – 1913	1914 – 1929	1930 – 1939 (NIPA data)	1914 – 1929/ *Prewar*
GNP Estimates				
Romer	2.07	2.77	5.62	1.34
Balke-Gordon	2.47	4.10	5.62	1.66
1867-1995 dataset, normalized to NIPA 1946-1995				
FACTOR 53 SERIES				
Constant	2.00	5.25	6.92	2.63
Time Varying	1.51	3.54	5.02	2.34
1867-1929 dataset, normalized to Balke-Gordon 1867-1929				
FACTOR 53 SERIES				
Constant	1.96	4.95		2.51
Time Varying	1.97	4.95		2.51
FACTOR 98 SERIES				
Constant	2.38	4.34		1.82
Time Varying	2.18	4.70		2.16
1867-1939 dataset, normalized to NIPA 1930-39				
FACTOR 53 SERIES				
Constant	1.67	4.27	5.62	2.56
Time Varying	1.82	4.38	5.62	2.41
FACTOR 98 SERIES				
Constant	1.75	4.25	5.62	2.42
Time Varying	1.95	4.62	5.62	2.37

Table 2.2 makes the outcome more explicit. The upper panel shows the standard deviation of the cyclical components in Romer's and Balke and Gordon's GNP estimates for subperiods up until 1929. As both series are spliced to the official NIPA series of GDP in 1929, the standard deviations of both series for 1930 to 1939 are identical. As before, the standard deviation of the factor estimates needs to be calibrated.

To do this, we choose three different approaches, each estimating the factors over a different time span. Under the first approach, the factor is estimated for the whole period to 1995 and its volatility calibrated to NIPA for 1946-1995. This is the same strategy adopted in Table 2.1 above. Results are shown in the second panel of Table 2.2. The second approach is to estimate the factor only from 1867 to 1929, and to calibrate to the cyclical component of the Balke and Gordon (1989) series. As we have more series available for this subperiods, we conduct this experiment twice, once for the same 53 series that are available through 1995, the second time for the wider dataset of 98 series. This strategy also underlies Figure 2.3. Results are shown in the center panel of Table 2.2. The third approach, shown in the lower panel of Table 2.2 is to estimate the factors from 1867 to 1939, and to calibrate to the standard deviation of NIPA for 1930 to 1939.

As the factor estimates are not recursive, truncation of the estimation period affects the results for all subperiods. Truncating to 1867-1929, which is the period of interest in this section, makes for an unbiased comparison of volatilities across World War I. Extending the estimation period to 1995, as in the upper panel, or to 1939, as in the lower panel, introduces potential bias but permits calibrating the factors to the volatility of the official NIPA data. As a consequence, volatility in the pre-1929 years can be directly compared to volatility in the NIPA series for relevant subperiods.

Three results stand out from this robustness check. First, the increase in factor volatility across World War I consistently comes out higher than in either Romer's or Balke and Gordon's GDP estimate (Table 2.2, last column). This result is robust to truncations of the estimation period, as well as to widening the database for the factor estimate from 53 to 98 series. It is also remarkably invariant to the choice between constant and time-varying factor loadings. The second main result is that pre-1914 volatility in the factor estimates is always lower than the Balke/Gordon estimate would suggest (Table 2.2, first column). For the most part, the factors even suggest lower business cycle volatility than implied by the Romer estimate. This effect also obtains in those factor estimates which are calibrated to NIPA, be it for the postwar period or for 1930 to 1939. In both cases, prewar volatility is close to the postwar level of volatility (2.01, see Table 1 above) and in many cases markedly lower. The third main result is that volatility during 1914 to 1929 (second column in Table 2.2) is consistently higher than estimated by Romer [1989], and is indeed close to or even higher than in the Balke and Gordon (1989) data.

This result has additional implications for evaluating the outcomes of the debate between Romer and Balke and Gordon. Under various robustness checks, we find there is no evidence of postwar moderation relative to the pre-1914 period. This would confirm a main point of Romer [1989]. On the other hand, we also find quite strong evidence of a marked volatility increases across World War I. This in turn would confirm a result of Balke and Gordon (1989) against criticism by Romer [1988].

2.3.3 The US Business Cycle Across World War II

Discrepancies between output and income based estimates of GDP exist also from 1929 onwards, when the NIPA accounts set in. These are themselves a compromise, leaning toward the results of the Commerce Department's earlier output series. The same alternative series produced by Kuznets [1961] and Kendrick [1961] that underlie much of Romer's (1986, 1988, 1989) GDP revisions for the pre-1929 period also show less volatility than the NIPA series of GDP for 1929 to 1945. This effect is particularly noticeable for the World War II years, where the income based estimates suggest a less pronounced increase in economic activity, as well as a different business cycle chronology.[9]

In the following, we zoom in on the years 1929 to 1949 and compare the official national accounting figures with the income-based estimate by Kuznets [1961]. The upper panel plots the factor against the official NIPA accounts. The income estimate of Kuznets [1961] is shown in the lower panel. Data are again detrended by a HP(6.25) filter.

The factor (thick blue line) is the factor from real 36 series obtained above in Table 1. Simple eye-balling quickly delivers the message: Until 1938 the business cycle turning points in the factor are very close to those of both NIPA and Kuznets' income estimate (in passing we note the earlier trough of the Great Depression implied by the factor). During the war, however, the factor tracks the Kuznets estimate much more closely than the official Commerce series on which the wartime NIPA data are based. According to our factor estimate, increasing wartime production did hardly offset the fall in civilian activity. In 1945, the lower turning point was reached by both measures.

The official NIPA data convey a different impression: from the lower turning point in 1940 on, they suggest an unprecedented rise in real output until 1944 – almost at the end of the war and one year before the factor and Kuznets' aggregate have their lower turning point. At the peak of war production, the economy fell into a deep recession

[9] For the discussion see Kuznets [1945], Mircgell [1943], Nordhaus and Tobin [1972], and a review in [Higgs, 1992, p. 45].

that lasted throughout the postwar years until 1949.

Kuznets' estimate is intended not only to account for the war related surge in commodity output but also to reflect consumption opportunities at the home front more accurately.[10] The real factor, which includes a number of broadly based business cycle indicators, may capture cyclical movements in the broader economy better than the commodity-based Commerce Department's estimate, which is again the point made by Romer [1989] for the years prior to World War I.[11]

This result is generally robust to widening the basis of the factor as well as including constant factor loadings. Including nominal series to arrive at the factor from 53 series employed elsewhere in this chapter, we arrived at a highly damped cyclical component for the wartime year, which shows even less volatility than the Kuznets estimate while again exhibiting the same turning points. We take this as further evidence supporting Kuznets's emphasis on properly deflating any nominal wartime series. We leave this as a subject for future research.

Summing up, World War II is the one period where our factor exhibits marked deviations from turning points in the the official NIPA figures. Dynamic factor models have proved to be excellent business cycle indicators in the presence of abundant data Stock and Watson [1998] as well as when data are scarce Gerlach and Gerlach-Kristen [2005]. The cyclical behavior of the factor appears to support Kuznets and others who called for a revision of the official historiography of the American business cycle during World War II.

2.4 Conclusions

Factor analysis of aggregate economic activity represents an appealing alternative and complement to Historical National Accounts whenever the data are incomplete or plagued by structural breaks in reporting. In this chapter, we re-examined the volatility of historical business cycles in the U.S. since 1867 using a dynamic factor model. Based on a large set of disaggregate time series, we obtained factors representing both aggregate and sectoral activity in the U.S. economy, and employed them to compare volatility

[10] Carson [1975] has more details on this debate.

[11] Another important point are reliable price indices, see Kuznets [1952], while the Kuznets-series presented here have been calculated with the same prices as the Commerce series for reasons of long-term comparability Kuznets [1961]. Further research, however, will take the price argument up again.

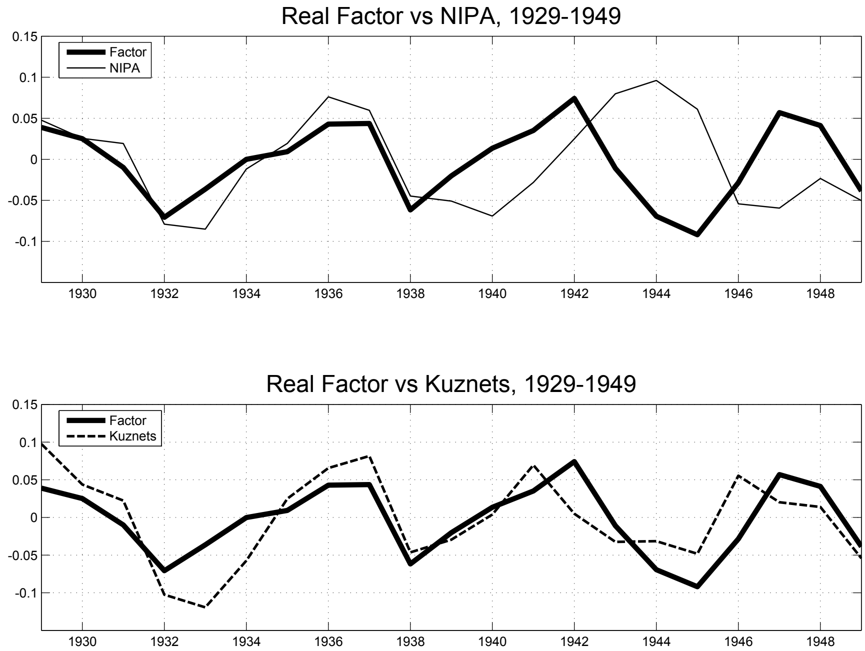

Figure 2.4: Factor from 36 real series vs. rivaling estimates of GNP during World War II. All data are deviations from HP(6.25) trend, FL-prior: 1/100; i.e. tight around 1%.

across World War I as well as in the long run.

Our main finding is that the business cycle prior to World War I may have even been less volatile than has previously been thought, and was quite plausibly no more volatile than the postwar business cycle. We also find pervasive evidence that the interwar years, in particular the period immediately following World War I, were more volatile than has been maintained in parts of the more recent literature. This would make the Great Depression of the early 1930s less of a historical singularity.

For the years surrounding World War II we find indications that the standard figures for national output misrepresent the business cycle turning points, and that both the wartime boom and the postwar bust of the US economy may have been weaker than suggested by the official NIPA data in GDP. These findings confirm earlier results by Kuznets [1961] and Kendrick [1961].

As would be expected, many of our results derive from the analysis of time variation in factor loadings, the weights assigned to the various individual series in constructing the index of aggregate economic activity. To this end, we employ a Bayesian approach to factor analysis, iterating over the likelihood function by Gibbs sampling. Our approach nests both constant and time-varying factor loadings. We slow time variation in the factor loadings to be an effective way of dealing with the structural changes in the U.S. economy, a problem that is hard to deal with in HNA approaches. Our findings suggest that spurious volatility in national accounts of the U.S. business cycle is to a large extent the consequence of time-invariant weighing schemes that underlie much work in national accounting with historical data.

Our findings are closely related to earlier work by Romer [1986, 1988, 1989] and Balke and Gordon [1989], which was based on backward extrapolations of national accounts into the late 19th and early 20th century. Balke and Gordon [1989] concluded from one standard GDP estimate that the U.S. business cycle was markedly more moderate in the postwar period than before the Gold Standard. Based on a rivaling estimate and imposing time-varying weighing schemes, Romer [1988, 1989] found little evidence of such postwar moderation. However, which is the better estimate remained open, as there appeared to be no way to validate the underlying assumptions independently. Our approach can be viewed as an attempt to provide such a validation method.

The flexibility of the estimation approach allowed us to recast the debate in terms of the factor model. Keeping factor loadings constant and thus shutting down structural change, we were able to reproduce the postwar moderation result. The same result also obtained when limiting attention to a subset of series representing material goods production, close in spirit to the Commerce Series of GDP employed by Balke and Gordon [1989]. On the other hand, when allowing for time varying factor loadings – and thus structural change –, our results were closer to Romer's (1989) and even more pronounced. Weaker but qualitatively similar results obtained when broadening the database to include other than material goods output. Hence, the identification assumptions used by these authors generate qualitatively similar results under a rather different methodology, a robustness property that we find remarkable. Given that the time varying model produces a better overall description of the postwar data and is also is more appealing on a priori grounds, we lean toward Romer's (1989) conclusion of no moderation in the postwar U.S. business cycle relative to pre-1914. However, time variation or a widening of the dataset do not in all cases explain the differences between the rivaling national account series. Our factor estimates invariably suggest a marked recession in 1920/21, which is borne out by the Commerce series in Balke and

Gordon (1989) but not by the Kuznets/Kendrick series in Romer [1988, 1989]. Postwar moderation does, however, obtain in the nominal data. A nominal factor becomes less volatile in the postwar era relative to pre-1914 if factor loadings are allowed to vary. With factor loadings fixed, however, we again arrive at the result of Balke and Gordon [1989]: less real postwar volatility, but substantially more nominal fluctuations.

3 Dynamic Index Models: A Bayesian Perspective

This chapter studies dynamic index models from a Bayesian perspective. The fact that the presence of a unit root is of no importance in Bayesian econometrics is exploited here. To determine the number of common components the predictions of competing model specifications are compared and therefore the Bayes factor is calculated. The model is applied to an artificial and a large-scale post WWII U.S. dataset, which is not transformed to eliminate trends and is used just in levels. The unobserved indices are estimated precisely and their number is determined correctly. A further advantage is that the method automatically decomposes the data into common trend and common cyclical components.

3.1 Introduction

It is a delicate issue to assess the exact state of the economy. Since there is no general agreement on the sources of economic fluctuations, choosing from the huge amount of available time series turns out to be complicated. To establish model-independent stylized facts, a parsimonious procedure, allowing to cope with a vast amount of information, would be desirable. Such a procedure is formalized in a dynamic index (factor) model, which exploits the comovement between economic series and therefore reduces the system to a set of aggregates. In recent years, this method has become very popular. Among other things, it has been extensively applied to extract economic indices and to improve the forecasting accuracy of economic variables. But, as in most of these studies, the presence of a unit root still necessitates special attention, as the classical asymptotic distribution theory for integrated series differs from that of stationary series. These difficulties evaporate when the same problem is approached from a Bayesian perspective.

In this chapter, a dynamic index model with stationary and nonstationary components is estimated using Bayesian methods. As emphasized in Sims [1988], Sims and Uhlig [1991], and Uhlig [1994], I exploit the fact that the stationarity of a series is of no importance in Bayesian econometrics. The case of a unit root is just one of many possibilities. Conditional on the data, the unit root case merely obtains a certain posterior weight. The model is estimated with the Gibbs sample algorithm in an efficient one-step estimation procedure. Moreover, to determine the number of factors[1] I rely on the comparison of predictions of the competing model specifications (e.g., Jeffreys, 1961). Thus, the Bayes factor[2] is calculated for the index model, using the procedure described in Chib [1995].

Dynamic index models decompose a given series into two unobserved orthogonal components - a common and an idiosyncratic component. They are first described in Sargent and Sims [1977], Geweke [1977], and Stock and Watson [1989]. The former two use a frequency domain approach, while the latter estimates the model in the time-domain. A more general approach, which combines the approximative but static factor model by Chamberlain and Rothschild [1983] with dynamic factor models is described in Forni, Hallin, Lippi, and Reichlin [2000]. Focusing on the application to large-scale - potentially nonstationary - data, Quah and Sargent [1993], Forni and Reichlin [1998], and Bai [2004] deal with factor models, which include common permanent and transitory components. However, only Bai [2004] provides rates of convergence and limiting distributions for the parameters and the common stochastic trends. An extension, where the idiosyncratic components are additionally allowed to be integrated is described in Bai and Ng [2004].

The method applied in this article can be seen as a Bayesian alternative to the likelihood-based approach of Quah and Sargent [1993] and the principal component approaches suggested by Forni and Reichlin [1998] and Bai [2004].[3] From a technical point of view, the Gibbs algorithm replaces the Expectation-Maximization (EM) algorithm, as it is utilized by Quah and Sargent [1993], and a fully parametric one-step procedure is used instead of the nonparametric approaches employed by Forni and Reichlin [1998] and Bai [2004]. Furthermore, working with Bayesian methods allows a straightforward derivation of the uncertainty around the factors and parameters. In particular, the parameter uncertainty might be important when it comes to forecasting economic series, as it may yield to misleadingly precise predictions (e.g., Uhlig, 1994, Reichmuth and Sarferaz,

[1] See Bai and Ng [2002] and Bai [2004] for a classical approach.
[2] When the prior on the competing model specifications are equal, the Bayes factor is equivalent to the posterior odds ratio.
[3] A Bayesian alternative for stationarized data is provided by, for instance, Kim and Nelson [1999a] and Otrok and Whiteman [1998].

2008b). Moreover, as the index model can be seen as just an approximation to reality, model uncertainty can be an important issue. Even though not conducted here, estimating model uncertainty is straightforward when the Bayes factor is readily available (e.g., Leamer, 1978, Kass and Raftery, 1995).

To illustrate the accuracy of the approach I first apply the model to an artificial dataset, which contains a common stochastic trend with drift and a common cycle. It can be observed that the procedure is able to distinguish between the common stationary and nonstationary components, without any restrictions on the dynamics of the model. All factors are estimated very precisely. Furthermore, the number of factors match exactly the number of factors specified for the artificial dataset. In a second step, the Bayesian dynamic index model is applied to post WWII U.S. data. The dataset has recently been compiled by Stock and Watson [2008] and consists of 108 macroeconomic time series. Opposed to Stock and Watson [2008], the data are analyzed in levels and are not transformed to eliminate trends. The Bayes factor for the Stock-Watson dataset is calculated to determine the number of factors. The procedure recommends to choose seven factors. Visual inspection reveals that the procedure is able to estimate the factors very precisely. Moreover, the model automatically decomposes the data into common trend and cyclical components, even though no restrictions on the parameter space of the model were imposed in this regard.

The rest of the chapter is organized as follows. Section 3.2 describes the dynamic index model and illustrates the corresponding identification problem. Section 3.3 introduces the prior distributions and Section 3.4 describes the estimation procedure. In Section 3.5 the Bayes factor for the dynamic index model is calculated. Section 3.6 describes the dataset for postwar U.S. and Section 3.7 summarizes the results, using the artificial data and the U.S. data. Finally, Section 3.8 concludes and discusses possible applications.

3.2 The Model

A dynamic index model implicates that an observed variable $x_{j,t}$ for $j = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, T$ can be expressed as

$$x_{j,t} = \alpha_j + \beta_j U_t + v_{j,t}, \tag{3.1}$$

where U_t is a $K \times 1$ vector of possibly stationary and nonstationary unobserved factors. The so called factor loadings in the $1 \times K$ coefficient vector β_j connect each increment of the vector of latent indices with the observed variable $x_{j,t}$. The constant term α_j is the arithmetic mean $\alpha_j = \frac{1}{T}\sum_{t=1}^{T} x_{j,t}$. Finally, $v_{j,t}$ is the variable specific component, which is assumed to be stationary, uncorrelated with the common components U_t at all leads and lags, and independent across all j.

The common components follow an VAR(q) process:

$$U_t = C + \Phi_1 U_{t-1} + \ldots + \Phi_q U_{t-q} + \nu_t, \tag{3.2}$$

with $\nu_t \sim \mathcal{N}(0, \Sigma_\nu)$. Thus, the integrated and the stationary components are correlated in this setup, which deviates from the models described in Quah and Sargent [1993], Forni and Reichlin [1998], and Bai [2004].

For the idiosyncratic components $v_{j,t}$ an AR(p) process is assumed:

$$v_{j,t} = \theta_{j,1} v_{j,t-1} + \ldots + \theta_{j,p} v_{j,t-p} + \eta_{j,t}, \tag{3.3}$$

where $\eta_{j,t} \sim \mathcal{N}(0, \sigma_{j,\eta})$.

The model stated in equations (3.1) - (3.3) is not uniquely identified. It suffers from sign, scale and rotational indeterminacy, which is illustrated briefly in the following. Multiplying equation (3.1) by $1 = \frac{a}{a}$ and $a \neq 0$ results in

$$x_{j,t} = \alpha_j + (a\beta_j)\left(\frac{U_t}{a}\right) + v_{j,t},$$

implicating the same likelihood as equation (3.1). A further - rotational - indeterminacy emerges, when

$$x_{j,t} = \alpha_j + (\beta_j P')\,(P U_t) + v_{j,t}\,, \tag{3.4}$$

where P is an orthogonal matrix. Again, equation (3.1) and equation (3.4) imply the same likelihood. To just identify the model I set the upper $K \times K$ block of β, where β contains all β_j's in stacked form, to a lower triangular matrix with ones on the diagonal (see, Geweke and Zhou, 1996, Bernanke, Boivin, and Eliasz, 2005, Moench, 2008).

3.3 Priors

In this section the prior distributions used for the dynamic index model in Section 3.2 are introduced. The prior distributions are without exception all proper, since the calculation of the marginal likelihood described in Section 3.5 requires proper distributions.[4]

The prior for the AR-parameters in equation (3.3) is described as

$$\theta^{prior} \sim \mathcal{N}(0, B_\theta),$$

where

$$B_\theta = \tau_1 \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \frac{1}{2} & \vdots & \vdots \\ \vdots & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{p} \end{bmatrix}.$$

The prior gets more tight around its zero mean for more distant lags, which is similar to the prior suggested in Doan, Litterman, and Sims [1984]. I impose stationarity on the idiosyncratic components through an indicator function which discards nonstationary draws. Additionally, assuming $\tau_1 = (0.01)^2$ the posterior of the AR-parameters are "pushed" closer to zero. This implies that we have a white noise prior on the AR-process, which is well suited for business cycle analysis.

The following prior for the factor loadings is used

$$\beta^{prior} \sim \mathcal{N}(0, B_\beta),$$

where $\underline{B}_\beta = 10$. The prior for the variance of the disturbances $\eta_{j,t}$ can be described as

$$\sigma_\eta^{prior} \sim \mathcal{IG}\left(\frac{\alpha_\eta}{2}, \frac{\delta_\eta}{2}\right)$$

[4]See, for example, Leamer [1978].

where $\alpha_\eta = 6$, $\delta_\eta = 0.01$, and $\mathcal{IG}$ is the inverted gamma distribution.

For the prior on the VAR parameters an Inverted Wishart-Normal prior is assumed. In addition, each variable is assumed to follow an univariate AR-process, where the mean of the parameters on the first lag is set to one and on the subsequent lags to zero. The uncertainty around these mean values decreases for higher order lags, implying that more distant lags are less important (e.g., Doan, Litterman, and Sims, 1984, Sims and Zha, 1998). The prior on the VAR parameters can be described as

$$\Sigma_{prior} \sim \mathcal{IW}(\tau_2 \mathcal{I}_K, K+2),$$

where $\tau_2 = 0.01$, with $\mathcal{IW}$ representing the inverse Wishart distribution and

$$vec(\Phi_{prior}) \sim \mathcal{N}(vec(\underline{\Phi}), \underline{\Omega}),$$

where $\underline{\Omega} \equiv \Sigma_{prior} \otimes B_\Phi$ and

$$\underline{\Phi} = \begin{bmatrix} \mathcal{I}_K \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \text{with} \quad B_\Phi = \begin{bmatrix} \tau_3 \frac{\mathcal{I}_K}{1} & 0 & \cdots & 0 \\ 0 & \ddots & \vdots & \vdots \\ \vdots & \cdots & \tau_3 \frac{\mathcal{I}_K}{q} & 0 \\ 0 & \cdots & 0 & \tau_4 \end{bmatrix},$$

where $\tau_3 = 0.2$, which is also called the *overall tightness* and $\tau_4 = 100$, which determines the uncertainty around the constant. The $K \times K$ identity matrix is denoted by $\mathcal{I}_K$.

3.4 Estimation

The Gibbs algorithm (Geman and Geman, 1984, Gelfand and Smith, 1990) allows to generate draws from the posterior, even when the joint posterior distribution is not tractable. Requiring a complete set of conditional densities, the Gibbs sampler proceeds as follows. Assume that Ψ contains all parameters of the model and U all latent variables. Given the initial values Ψ^0, U^0, which are arbitrary chosen, $\{\Psi^1 \sim p(\Psi|U^0), U^1 \sim p(U|\Psi^1)\}$, $\{\Psi^2 \sim p(\Psi|U^1), U^2 \sim p(U|\Psi^2)\}$, ... , $\{\Psi^w \sim p(\Psi|U^{w-1}), U^w \sim p(U|\Psi^w))\}$ is drawn, leading to the Gibbs sequence $\{\Psi^w, U^w\}$. It turns out that under mild conditions the Gibbs sequence $\{\Psi^w, U^w\}$ converges (in distribution) to the true joint density at a geometric rate in w (Geman and Geman, 1984).[5] For a more detailed discussion of the estimation procedure I refer to the Appendix of this chapter.[6]

3.5 The Bayes Factor

To calculate the Bayes factor is usually not an easy task.[7] The difficulty is that the marginal density of the data is needed as input, which requires that the probability density function has to be integrated over the parameter space. One way to solve this problem is to apply the Laplace approximation (e.g., Lindley, 1980, Tierney and Kadane, 1986). However, for complex models the Laplace method turns out to be inaccurate.

The application of simulation-based methods, constitutes another and even more exact way to calculate the marginal likelihood. In order to compute the marginal density of the data from the Gibbs output, Newton and Raftery (1994) propose a procedure, which takes the harmonic mean over all likelihood values. However, as the inverse of the likelihood does not have a finite variance, the method is prone to instability. A modification suggested by Gelfand and Dey (1994) uses a tuning function. Admittedly, to find the appropriate tuning function is sometimes hard to do, especially when the dimension of the model is large.

Chib [1995] suggests a procedure, which exploits the fact that the marginal likelihood is equivalent to the normalizing constant. The method does not depend on the inverse of the likelihood and thus does not struggle with instability problems. Moreover, it works

[5] Note that Ψ^w is itself a Gibbs sequence, cycling over the parameter space. See Section 3.5 for further details.

[6] See also Kim and Nelson [1999a] and Eliasz [2002].

[7] For an overview see, among others, Kass and Raftery [1995] and Han and Carlin [2001].

accurately for high-dimensional systems and is designed to incorporate latent variables. Thus, it is adequate for the dynamic index model used here.[8] Following Chib [1995], the marginal likelihood of a model M_k, for $k = 1, \ldots, K_{max}$ is defined as

$$m(y|M_k) = \frac{p(y|\Psi, M_k)\pi(\Psi|M_k)}{p(\Psi|y, M_k)}, \tag{3.5}$$

where $p(y|\Psi, M_k)$ is the likelihood function, $\pi(\Psi|M_k)$ is the prior density, and $p(\Psi|y, M_k)$ is the posterior density. In the following I drop for notational convenience the model classifications M_k. Taking logs and evaluating equation (3.5) at a high density point Ψ^*, say the posterior mean, results into the following expression

$$ln\ m(y) = ln\ p(y|\Psi^*) + ln\ \pi(\Psi^*) - ln\ \hat{p}(\Psi^*|y). \tag{3.6}$$

To obtain the likelihood function $p(y|\Psi^*)$ and the prior density $\pi(\Psi^*)$ evaluated at Ψ^* is straightforward. Following Chib [1995] it is assumed that the conditional densities of the Gibbs procedure can be expressed as

$$p(\psi_1|\psi_2, \ldots, \psi_B, z, y),\ p(\psi_2|\psi_1, \psi_3, \ldots, \psi_B, z, y),\ \ldots\ , p(\psi_B|\psi_1, \ldots, \psi_{B-1}, z, y)$$

and

$$p(z|\psi_1, \psi_2, \ldots, \psi_B, y),$$

where z represents a vector of latent variables, and $\Psi \equiv \{\psi_1, \psi_2, \ldots, \psi_B\}$ represents the parameters of the model. The output of the Gibbs algorithm is defined as $\{\psi_1^{(w)}, \psi_2^{(w)}, \ldots, \psi_B^{(w)}, z^{(w)}\}$ for $w = 1, 2, \ldots, W$. The marginal posterior density of the parameters $p(\Psi^*|y)$ evaluated at Ψ^* is

$$p(\Psi^*|y) = p(\psi_1^*|y) \times p(\psi_2^*|\psi_1^*, y) \times \ldots p(\psi_B^*|\psi_1^*, \psi_2^*, \ldots, \psi_B^*, y) \tag{3.7}$$

[8] A similar procedure, which uses the methods described in Chib [1995] and Chib and Jeliazkov [2001] in combination, is described in Otrok, Silos, and Whiteman [2003].

where

$$
\begin{aligned}
p(\psi_1^*|y) &= \int p(\psi_1^*|\psi_2,\ldots,\psi_6,z,y)p(\psi_2,\ldots,\psi_B,z|y)d\psi_2,\ldots,d\psi_B dz,\\
p(\psi_2^*|y) &= \int p(\psi_2^*|\psi_3,\ldots,\psi_B,z,\psi_1^*,y)p(\psi_3,\ldots,\psi_B,z|\psi_1^*,y)d\psi_3,\ldots,d\psi_B dz,\\
&\vdots\\
p(\psi_B^*|y) &= \int p(\psi_B^*|z,\psi_1^*,\psi_2^*,\ldots,\psi_B^*,y)p(z|\psi_1^*,y)dz.
\end{aligned}
$$

Using the initial Gibbs run $p(\psi_1^*|y)$ can be estimated with the ergodic average

$$\hat{p}(\psi_1^*|y) = \frac{1}{W}\sum_{w=1}^{W} p(\psi_1^*|\psi_2^{(w)},\psi_3^{(w)},\ldots,\psi_B^{(w)},z^{(w)},y). \tag{3.8}$$

To estimate $p(\psi_2^*|y), p(\psi_3^*|y),\ldots,p(\psi_B^*|y)$ it is required to rerun the Gibbs sampler, while conditioning on the values calculated in the previous runs. To estimate $p(\psi_2^*|y)$, the conditional densities

$$p(\psi_2|\psi_1^*,\psi_3,\ldots,\psi_B,z,y),\ \ldots\ ,p(\psi_B|\psi_1^*,\psi_2,\ldots,\psi_{B-1},z,y),p(z|\psi_1^*,\psi_2,\ldots,\psi_B,y)$$

can be used to obtain

$$\hat{p}(\psi_2^*|y) = \frac{1}{W}\sum_{w=1}^{W} p(\psi_2^*|\psi_3^{(w)},\psi_4^{(w)},\ldots,\psi_B^{(w)},z^{(w)},\psi_1^*,y). \tag{3.9}$$

The posterior ordinate $p(\psi_3^*|y)$ is calculated using further draws from

$$p(\psi_3|\psi_1^*,\psi_2^*,\psi_4,\ldots,\psi_B,z,y),\ \ldots\ ,p(\psi_B|\psi_1^*,\psi_2^*,\psi_3,\ldots,\psi_{B-1},z,y),$$

$$p(z|\psi_1^*,\psi_2^*,\psi_4,\ldots,\psi_B,y),$$

which gives

$$\hat{p}(\psi_3^*|y) = \frac{1}{W}\sum_{w=1}^{W} p(\psi_3^*|\psi_4^{(w)},\ldots,\psi_B^{(w)},z^{(w)},\psi_1^*,\psi_2^*,y). \tag{3.10}$$

The final conditional densities are

$$p(\psi_B|\psi_1^*,\psi_2^*,\ldots,\psi_{B-1}^*,z,y),p(z|\psi_1^*,\psi_2^*,\ldots,\psi_{B-1}^*,\psi_B,y)$$

resulting in

$$\hat{p}(\psi_B^*|y) = \frac{1}{W}\sum_{w=1}^{W} p(\psi_B^*|z^{(w)},\psi_1^*,\psi_2^*,\ldots,\psi_{B-1}^*,y). \tag{3.11}$$

Plugging equations (3.8)–(3.11) into (3.6) leads to the following estimate of the marginal likelihood

$$\ln\ \hat{m}(y) = \ln\ p(y|\Psi^*) + \ln\ p(\Psi^*) - \sum_{i=1}^{B} \ln\ \hat{p}(\psi_i^*|y)$$

The Bayes factor in favor of model M_k can now be computed as

$$B_{k,l} = \exp\{\ln\ \hat{m}(y|M_k) - \ln\ \hat{m}(y|M_l)\}.$$

For $\Psi \equiv \{\beta_j, \theta_{j,1}, \theta_{j,2}, \ldots, \theta_{j,p}, \Phi_1, \Phi_2, \ldots, \Phi_q, \Sigma_\nu, \sigma_\eta\}$ and $z \equiv [U_1\ U_2\ \ldots\ U_T]$ the procedure is applicable to the dynamic index model described in Section 3.2.

3.6 Data

The dataset used here is compiled by Stock and Watson [2008]. It consists of 108 quarterly series, covering a broad range of disaggregated macroeconomic series. The dataset contains real variables such as sector specific industrial production and employment, and nominal series such as sector specific price indices, interest rates, and money supply measures. While Stock and Watson [2008] apply several transformations to stationarize their data, the data analyzed here are in levels. Thus, the data span 1959:I–2006:IV, implying $T = 192$ observations (Stock and Watson have $T = 190$ observations available). The data are standardized.[9]

3.7 Empirical Results

The Gibbs algorithm produces 15,000 draws, the first 5,000 are discarded as burn-in. The number of additional draws for the calculation of the marginal likelihoods are set to 2,000 in each case. For both datasets, the simulated and the Stock-Watson dataset, convergence diagnostics are conducted. First, I start repeatedly from different overdispersed starting values and observe that the sampler already converges after a few thousand draws. Second, I compare the first half with the second half of the sampler. Both halves show strong similarities.[10]

[9] For a more detailed description of the dataset, see Table A.1 in Stock and Watson [2008].
[10] Trace plots are available on request.

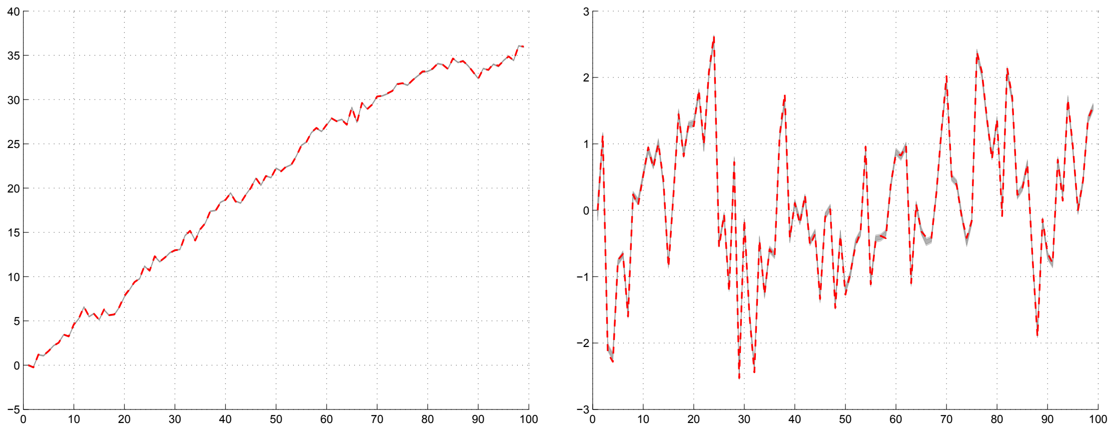

Figure 3.1: Error bands for the factors (gray shaded area) compared to the simulated factors (red dotted lines). The gray shaded area covers 90% of the posterior probability mass.

3.7.1 Results for Simulated Data

The data are simulated using

$$x_{j,t} = \beta_j U_t + v_{j,t},$$

$$U_t = C + \Phi_1 U_{t-1} + \Phi_2 U_{t-2} + \nu_t,$$

$$v_{j,t} = \theta v_{j,t-1} + \eta_{j,t},$$

where

$$C = \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix}, \; \Phi_1 = \begin{bmatrix} 0.6 & 0.001 \\ 0.001 & 0.3 \end{bmatrix}, \; \Phi_2 = \begin{bmatrix} 0.4 & 0.001 \\ 0.001 & 0.1 \end{bmatrix},$$

and $\theta = 0.2$. The factor loadings, which are assumed to be uniformly distributed are drawn from $\beta_j \sim U(0,1)$. The disturbances, which are assumed to be i.i.d. normal are drawn from $\eta_{j,t} \sim N(0, 0.1)$ and

$$\nu_t \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \right).$$

The dataset contains one common transitory and one common permanent component, hence, $K = 2$. The number of variables is $N = 60$ and the length of the time series $T = 100$.

Figure 3.1 plots the error bands for the factor estimates against the "true" factors. As can be seen, the Bayesian dynamic index model works accurately; the common stationary and nonstationary component are estimated with extremely tight error bands.

The estimated parameter and the population values for the artificial dataset are disputed in Table 3.1. For illustration purposes, the cross section averages of the parameters, dependent on the size of the cross section are presented. All parameters are estimated relatively precisely. However, the parameters for the law of motion of the idiosyncratic components are biased downwards, which is probably due to the very tight prior on the autoregressive parameters. Altogether, the results for the artificial dataset clearly show that the method used in this chapter works accurately.

		Posterior distribution	
	Pop. Value	Mean	Std. Dev.
β^1	0.51	0.51	0.001
β^2	0.58	0.59	0.01
σ^2	0.10	0.01	0.002
θ	0.10	-0.12	0.01
$\Phi^1_{1,1}$	0.60	0.63	0.08
$\Phi^1_{1,2}$	0.001	0.03	0.06
$\Phi^1_{2,1}$	0.001	0.11	0.11
$\Phi^1_{2,2}$	0.30	0.40	0.1
$\Phi^1_{1,1}$	0.40	0.36	0.08
$\Phi^1_{1,2}$	0.001	0.04	0.06
$\Phi^1_{2,1}$	0.001	-0.01	0.11
$\Phi^1_{2,2}$	0.10	0.13	0.09
$\Sigma_{\nu,1,1}$	0.50	0.40	0.06
$\Sigma_{\nu,1,2}$	0.00	0.04	0.06
$\Sigma_{\nu,2,1}$	0.00	0.04	0.06
$\Sigma_{\nu,2,2}$	1.00	0.87	0.12

Table 3.1: Population values and posterior distributions for artificial dataset. The cross section averages are defined as: $\beta^k = \frac{1}{N}\sum_{i=1}^{N}\beta_{i,k}$ for $k = 1, 2$, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}\sigma_i^2$ and $\theta = \frac{1}{N}\sum_{i=1}^{N}\theta_i$.

The results for the calculations of the Bayes factor can be found in Table 3.2. All models with up to $Kmax = 8$ are compared pairwise with each other. The dynamic index model with $K = 2$, corresponding to the true number of factors, is favored. This indicates that the Bayes factor procedure to determine the number of factors works properly.

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
M_1		0	∞	∞	∞	∞	∞	∞
M_2			∞	∞	∞	∞	∞	∞
M_3				∞	∞	∞	∞	∞
M_4					∞	0	∞	∞
M_5						0	∞	∞
M_6							∞	∞
M_7								0

Table 3.2: Computing the Bayes factor for the simulated dataset in favor of model M_k for $k = 1, 2, \ldots, 8$ with $B_{k,l} = \exp\{\ln\ \hat{m}(y|M_k) - \ln\ \hat{m}(y|M_l)\}$

3.7.2 Results for the Stock-Watson Dataset

For the Stock and Watson [2008] dataset, a lag-length of $q = 5$ for the factors is and $p = 1$ for the idiosyncratic components is chosen. To determine the number of factors I use a maximum number of $Kmax = 10$. I checked convergence of the Gibbs sampler similar to approach described in Section 3.7.1 and observed that the sampler already converged after few thousand draws.

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}
M_1		0	0	0	0	0	0	0	0	0
M_2			∞	∞	0	0	0	0	0	0
M_3				0	0	0	0	0	0	0
M_4					0	0	0	0	0	0
M_5						0	0	0	0	0
M_6							0	0	0	0
M_7								∞	∞	∞
M_8									0	∞
M_9										∞

Table 3.3: Computing the Bayes factor for the Stock-Watson dataset in favor of model M_k for $k = 1, 2, \ldots, 10$ with $B_{k,l} = \exp\{\ln\ \hat{m}(y|M_k) - \ln\ \hat{m}(y|M_l)\}$

Table 3.3 contains the calculated Bayes factors. It compares all model specifications (deviating only in the number of factors) from one to ten factors. The kth row represents the Bayes factor in favor of model M_k, where large values implicate that the model should be favored. Thus, according to Table 3.3 seven factors are favored, which are plotted in Figure 3.2. As can be seen, all factors are estimated precisely. Moreover, Figure 3.2 implicates that the dynamic index model decomposes the data into common one trend component and several common cyclical components.

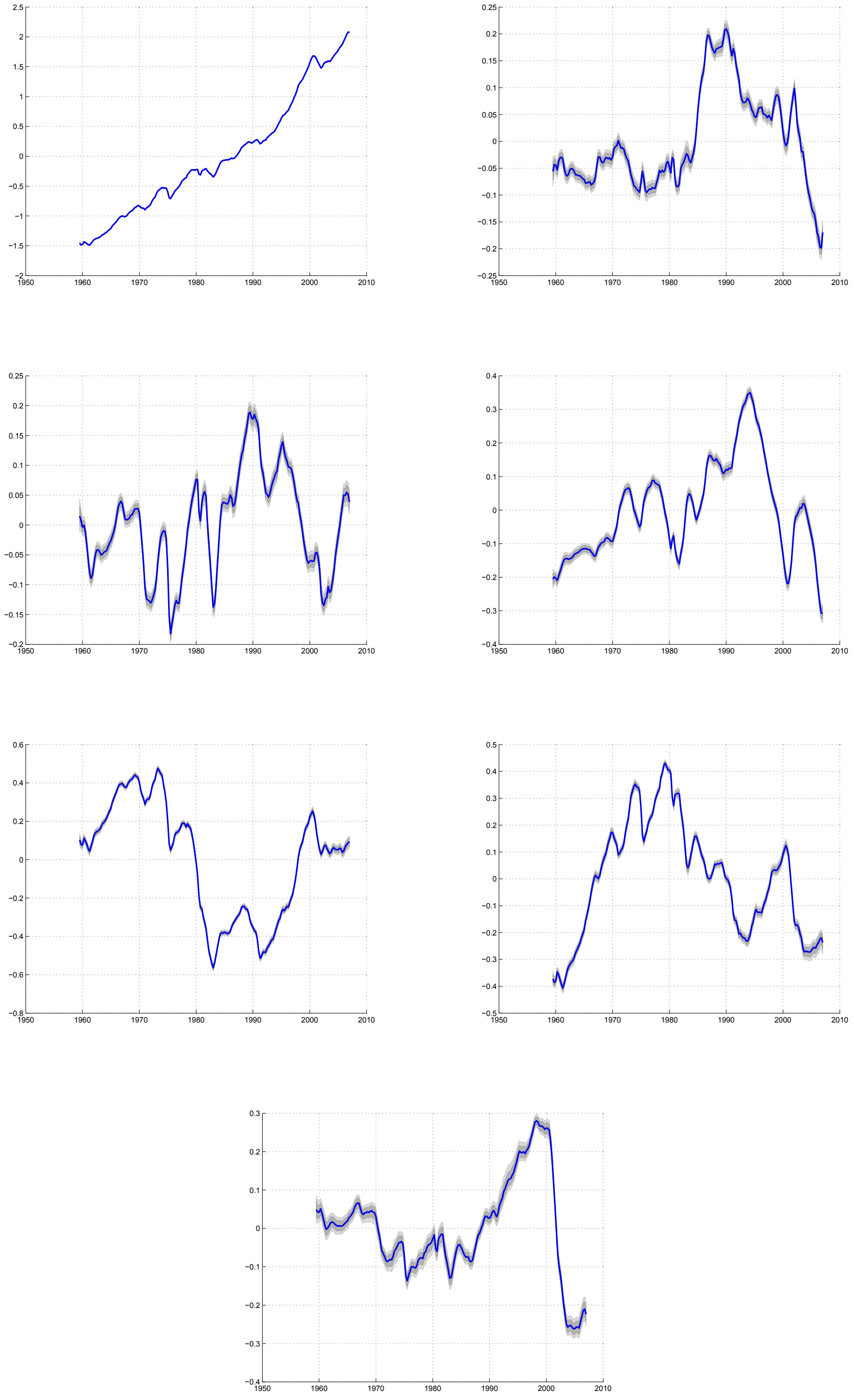

Figure 3.2: Estimated unobserved stationary and nonstationary common components of the Stock and Watson [2008] dataset. The dark gray shaded area represents 68% and the light shaded area 90% of the posterior probability mass.

3.8 Conclusion

This chapter studies dynamic index models from a Bayesian perspective. The fact that in the Bayesian approach the presence of a unit root is of no importance is used and is applied to a dynamic index model. Inference on datasets with large cross sections can hence be conducted, using the data in levels, which is appealing as no information is lost due to transformation of the data. Furthermore, model uncertainty and uncertainty surrounding factor and parameter estimates can be derived naturally. The model is estimated in a simulation-based one-step procedure, applying the Gibbs sampling algorithm. The number of unobserved common components is determined by comparing the predictions of competing model specifications, using the Bayes factor.

The model is tested with an artificial dataset, which contains a common stochastic trend with drift and common cycle. The procedure used in this chapter is able to distinguish between the common stochastic trend and the common cycle. The simulated stationary and nonstationary factors and the corresponding parameters of the artificial data are estimated precisely. Analyzing the artificial dataset also reveals that, determining the number of unobserved indices, using the Bayes factor method, works accurately. For the Stock and Watson [2008] dataset, where all series are used in levels, again the procedure was able to distinguish between common trend and common cycle components. All the corresponding uncertainty concerning the stationary and nonstationary factors and the parameters are provided as well.

The approach described in this chapter can be of use in many fields of macroeconomic research. One particular example is macroeconomic forecasting, as large-scale datasets can be used in levels, which avoids the loss of possibly important information. Characterizing the uncertainty surrounding the parameters and the model can be conducted in a coherent way. Another field of application of the procedure described in this chapter is the identification of macroeconomic shocks. The effects of aggregated shocks on more disaggregated series, e.g. monetary policy shocks, (see Bernanke, Boivin, and Eliasz, 2005) can be analyzed in a much less restrictive way, as the procedure does not require any pre-assumption on the persistence of the series.

4 Modeling and Forecasting Age-Specific Mortality

We present a new way to model age-specific demographic variables using the example of age-specific mortality in the United States. We build on the Lee–Carter approach and extend it in several dimensions. We incorporate covariates and model their dynamics jointly with the latent variables underlying mortality of all age classes. In contrast to previous models, a similar development of adjacent age groups is assured, allowing for consistent forecasts. We develop an appropriate Markov chain Monte Carlo algorithm to estimate the parameters and the latent variables in an efficient one-step procedure. Via the Bayesian approach we are able to assess uncertainty intuitively by constructing error bands for the forecasts. We observe that in particular parameter uncertainty is important for long-run forecasts. This implies that existing forecasting methods, which ignore certain sources of uncertainty, may yield misleadingly sure predictions. To test the forecast ability of our model we perform in-sample and out-of-sample forecasts up to 2050 revealing that covariates can help improve the forecasts for particular age classes. A structural analysis of the relationship between age-specific mortality and covariates is conducted in a companion paper.

4.1 Introduction

Demographic issues are of general interest since they address the most fundamental attributes of human life. Their research takes place at the crossways of economics and sociology, medicine, and other academic disciplines, which in turn are often influenced themselves by demographic findings. This gives rise to a multidisciplinary scientific interest. Of course, such research is not only of interest to science, but also to many recipients in the domains of politics and business. Reliable forecasts of future mortality and a better understanding of the determinants of changing mortality are obviously of great importance in areas such as social security and public health. In the private sector such advancements of knowledge can have substantial monetary value, since they improve the calculation of life insurance rates and pension schemes for the insurance industry. Population forecasts that can be derived from demographic rates are another example of interest beyond pure science owing to their implications for investment decisions in the public and private sectors. All of these potential recipients benefit most from stochastic models, which yield distributional statements on the probabilities of outcomes instead of pure projections of some scenarios. For this purpose, stochastic models of age-specific mortality and other demographic variables are needed.

We present a new way to model age-specific demographic variables using the example of age-specific mortality. Existing parametric and nonparametric approaches to modeling and forecasting mortality suffer from different shortcomings in the embodiment of the age dimension. Our model avoids these drawbacks. Furthermore, it is very general and comprises both the well-known Lee–Carter model and the use of covariates as special cases. Advanced methods from the domain of Bayesian time series econometrics are used to set up the model and estimate the parameters. Unobserved or latent variables, which drive the common development of the observed age-specific variables, are complemented by observable covariates. We formulate two explicit laws of motion in the form of (vector) autoregressions (VARs), which ensure a relatively smooth development not only along the time but also the age dimension of the demographic variable. For the latter, this is usually neglected. The importance of this issue is demonstrated by the very smooth surface without jumps in Figure 4.1 representing U.S. mortality. We feel confident that a reasonable model of age-specific mortality should explicitly embody this feature and guarantee such smoothness across ages in forecasts too. By the use of VARs we also allow for mutual interactions between latent variables and all covariates in the model. Finally, we use Markov chain Monte Carlo (MCMC) methods to estimate the model with an efficient one-step procedure. By the choice of priors this Bayesian estimation approach also clearly reveals the assumptions made. Most notably, it yields not only point estimates but also distributional statements for the results in a very intuitive way.

Our approach is very flexible and can be applied to model all kinds of demographic variables, using different numbers of latent variables and different sets of covariates. In this chapter, we present applications to U.S. mortality, with gross domestic product (GDP) and unemployment as important macroeconomic variables. Owing to our particular modeling approach, stochastic forecasts of the modeled variables are easily achieved and have the advantage of being fully consistent among adjacent age classes, unlike some parametric approaches or the popular Lee–Carter method. In addition to

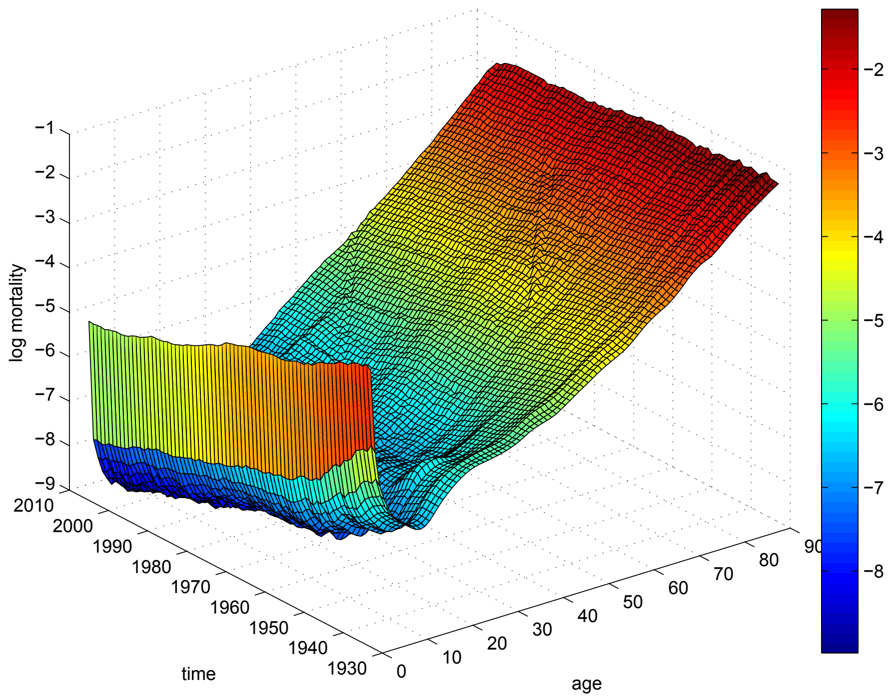

Figure 4.1: Mortality surface of the logarithmized age-specific total (female and male combined) mortality in the United States, 1933–2005.

this important feature of age-related smoothness, we also can distinguish the impact of different sources of uncertainty on the forecast results. We show that the uncertainty associated with the random terms in the model is more important at the beginning, whereas the uncertainty associated with the estimation of parameters is very important in a longer perspective. This means that false confidence in forecasts may result from ignoring important sources of uncertainty by concentrating on the random term, such as in the Lee–Carter model. In-sample forecasts reveal that both versions of the model, either including covariates or not, perform accurately. We present out-of-sample forecasts of mortality with respective error bands for a longer horizon up to the year 2050 which show that covariates can help improve the forecasts for particular age classes. Moreover, the use of VARs, which is facilitated by the enormous reduction of the dimension with the help of latent variables, allows for further structural analyses of the interactions between the covariates and the demographic variable, revealing the full pattern of age-specific reactions to external influences.

The presented approach can be applied to model, forecast, and analyze all kinds of age-specific variables. Mortality is just a prominent example owing to its great importance in general and to the fact that our model can be interpreted as a generalization of the established Lee–Carter model. Moreover, in addition to their own intrinsic value, forecasts of mortality also constitute an important part of the input needed for stochastic population forecasts with the cohort component method of stepwise interpolation of an initial population.

The rest of the chapter is organized as follows: Section 4.2 provides a brief summary of the literature on modeling and forecasting mortality. Our model is stated in Section 4.3. Section 4.4 describes the predictive densities. Sections 4.5 and 4.6 address the priors and the estimation procedure, and the data are described in Section 4.7. The estimation and forecast results are presented in Section 4.8, which additionally provides some intuitively interpretable life table variables based on age-specific mortality. Finally, Section 4.9 presents our conclusions.

4.2 Literature on Modeling and Forecasting Mortality

We start with a short overview of some developments in modeling and forecasting age-specific mortality.[1] Models that map age to age-specific mortality take advantage of the obvious strong regularities in mortality's age pattern.[2] In the context of forecasting, these regularities have to be taken into account, because naive univariate forecasts of each age-specific time series separately would propagate too much noise, quickly leading to serious inconsistencies. Of course, such models also substantially reduce the dimensionality of the data to be handled.

4.2.1 Parametric Modeling of Age-Specific Mortality

Systematic patterns in mortality have been known since the development of the first life tables by Graunt [1662] and Halley [1693]. In terms of a mathematical *law of mortality* for the observed age pattern, Gompertz [1825] first mentioned that mortality $m(x)$ at age x in adulthood shows a nearly exponential increase

$$m(x) = \alpha e^{\beta x}\,.$$

[1] Of course, we can only briefly sketch some major issues. Booth [2006] gives a comprehensive survey of demographic forecasting.

[2] For the sake of simplicity, except for the final life table calculations, we use the term *age-specific mortality* for both the probability ${}_1q_x = (l_x - l_{x+1})\,/\,l_x$ of dying at age x, which is related to the population at risk, that is, the number l_x of survivors to age x, and the death rate ${}_1m_x = (l_x - l_{x+1})\,/\,{}_1L_x$ at age x, which is related to the person-years ${}_1L_x$ lived at age x ($l_{x+1} \leq {}_1L_x \leq l_x$).

Among the many more sophisticated proposals for a formula of age-specific mortality since that time, Heligman and Pollard [1980] suggest a sum of three terms representing different components of mortality,

$$m(x) = A^{(x+B)^C} + De^{-E(\ln x - \ln F)^2} + GH^x / (1 + GH^x) \ ,$$

with eight time-dependent parameters $A_t, \ldots, H_t$. The rapidly falling first term accounts for mortality during childhood, the second term models the *accident hump* for young adults, and the third term picks up the Gompertz exponential for the senescent mortality of adulthood and old age. McNown and Rogers [1989] forecast the eight parameters of the Heligman–Pollard model using the univariate time series method of autoregressive integrated moving average (ARIMA) processes, which may lead to inconsistencies in the long run.

4.2.2 Lee–Carter and Non-parametric Modeling of Age-Specific Mortality

Non-parametric approaches to modeling age-specific mortality span from early model life tables to the nowadays well-established method of Lee and Carter [1992]. After the first set of model life tables released by the United Nations [1955], Coale et al. [1966] developed a two-dimensional set of four regional patterns, each with 24 different mortality levels identified by the life expectancy of children. Brass [1971] presents a relational model that maps a tabulated standard age pattern of mortality with two parameters to actual mortality.

Lee and Carter [1992] apply principal component analysis and propose a model

$$\ln (m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}$$

with mortality $m_{x,t}$ at age x and time t, fixed age effect a_x equal to the average observed log death rate, and an age-specific impact b_x of a time-specific general mortality index k_t. This single parameter k_t maps the average age pattern of mortality deviation from a_x to the actual pattern and b_x is the first principal component and is estimated by singular value decomposition. The subsequent estimation of the mortality index k_t as an ARIMA process results in a simple random walk with drift. The outcome, however, of forecasting age-specific mortality by this method with one time-dependent parameter is similar to that if each age-specific time series were extrapolated along its own historic time trend, potentially leading to an implausible age pattern in the long run.[3] This disadvantage is especially severe if the Lee–Carter approach is applied to single-cause mortality, for which it was not indeed assigned.[4] Nevertheless, the Lee–Carter method and its several enhancements have become the standard for mortality forecasts and have also been used for the newly emerged stochastic population forecasts since Lee and Tuljapurkar [1994] and Lee [1998].

There is broad literature introducing models more or less similar to the Lee–Carter approach. Lee [2000] reviews the original model as well as some of its problems and extensions. Quantitative comparisons of several recent models are given by Cairns et al. [2007, 2008] but they only apply data for the age classes 60–89, that is, model a relatively even part of the full pattern of age-specific mortality, which is of course of special interest for the insurance industry. Renshaw and Haberman [2006] include an additional cohort effect estimated in a two-step procedure. To overcome potential roughness De Jong and Tickle [2006] apply smoothing along the age dimension by restricting the impact of several k_t on particular age classes with a spline matrix.[5] Delwarde et al. [2007] apply smoothing with a roughness penalty for both the Lee–Carter and a Poisson log-bilinear model.

Pedroza [2006] applies Bayesian methodology to mortality forecasting and adopts it to a state space reformulation of the Lee–Carter model. Girosi and King [2008] also generalize the Lee–Carter method to an analysis with several principal components instead of considering only the first one. Nevertheless, they advocate a completely different approach and run Bayesian regressions on socio-economic time series as explanatory covariates for mortality. Their main purpose is to establish a formalized way to incorporate additional information about regularities along a cross-section dimension of mortality, which may comprise age, sex, country, or cause of death, and generate priors to express experts' assessments of these similarities.

[3] This critique goes back to McNown [1992] and Alho [1992].
[4] Girosi and King [2008, pp. 38–42] discuss this point and give examples.
[5] In a different approach of a generalized linear model with Poisson errors, Currie et al. [2004] apply smoothing along both the age and time dimensions with splines and handle future values to be forecasted as missing values which are estimated simultaneously.

4.3 A Bayesian State Space Model

The dynamics of age-specific demographic variables can be captured by models based on a latent common component as in Lee and Carter [1992]. We follow this line of research and extend these models by including additional macro variables as covariates and relating them with the latent variable by a VAR. We assume an autoregressive (AR) process for the coefficients, which link the explanatory variables with the age-specific demographic variables, to ensure smoothness along the age dimension. For the estimation of this state space model we use Bayesian methods, providing an appropriate MCMC algorithm.

4.3.1 General Model

Given an observed demographic variable $d_{x,t}$ with age classes $x = 0, \ldots, A$ and time periods $t = 1, \ldots, T$, we can formulate the equation

$$d_{x,t} \;=\; \overline{d}_x + \beta_x z_t + \epsilon^d_{x,t} \tag{4.1}$$

with arithmetic mean $\overline{d}_x = \frac{1}{T}\sum_{t=1}^{T} d_{x,t}$ and explanatory variables $z_t \equiv [\kappa_t \quad Y_t]'$, where κ_t is a $K \times 1$ vector of unobservables and Y_t is an $N \times 1$ vector of observed covariates. The corresponding coefficient vector $\beta_x \equiv [\beta^\kappa_x \quad \beta^Y_x]$ is $1 \times M$, where β^κ_x is a $1 \times K$ vector and β^Y_x is a $1 \times N$ vector with $M = K + N$. We assume z_t and β_x follow vector autoregressive processes,

$$z_t = c + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \cdots + \phi_p z_{t-p} + \epsilon^z_t\,, \tag{4.2}$$

$$\beta_x = \alpha_1 \beta_{x-1} + \alpha_2 \beta_{x-2} + \cdots + \alpha_q \beta_{x-q} + \epsilon^\beta_x\,, \tag{4.3}$$

where c is an $M \times 1$ vector of constants, $\phi_1, \ldots, \phi_p$ are $M \times M$ matrices, and $\alpha_1, \ldots, \alpha_q$ are $M \times M$ diagonal matrices. We assume $\epsilon^d_{x,t} \sim i.i.d.\ \mathcal{N}(0, \sigma^2_d)$ for the disturbances in Equation (4.1), $\epsilon^z_t \sim i.i.d.\ \mathcal{N}(0, \Sigma_z)$ for the disturbances in Equation (4.2), and $\epsilon^\beta_x \sim i.i.d.\ \mathcal{N}(0, \Sigma_\beta)$ for the disturbances in Equation (4.3), where the covariance matrix Σ_β is a diagonal matrix. Thus each component of β_x in fact follows an autoregressive process on its own. All disturbances are assumed to be independent of each other.

4.3.2 Special Case Lee–Carter

To give a more intuitive introduction to our model, we will show in the following that the Lee–Carter model can be seen as a special case of our model. We begin by assuming that $z_t \equiv \kappa_t$, dropping Equation (4.3), and specifying an extremely strong prior on $\phi_1, \phi_2, \ldots, \phi_q$, where we specify the prior on ϕ_1 very tightly around 1 and the prior on $\phi_2, \ldots, \phi_q$ very tightly around 0. Of course, this can be applied by subsequently strengthening the power of the priors. For the extreme case, when the priors are very dominant, information emerging from the data will be completely ignored for the VAR parameters $\phi_1, \phi_2, \ldots, \phi_q$ and we obtain, approximately, the model

$$d_{x,t} \;=\; \overline{d}_x + \beta_x^{\kappa} \kappa_t + \epsilon_{x,t}^{d} \tag{4.4}$$

with an AR process for the mortality index κ_t,

$$\kappa_t = c + \kappa_{t-1} + \epsilon_t^{\kappa} \,, \tag{4.5}$$

which is the Lee–Carter model in state space representation as described in Pedroza [2006].

4.3.3 Augmenting the Simple Model with Covariates

The inclusion of covariates may noticeably improve the forecasts of demographic models.[6] Respective time series provide additional information, which is ignored otherwise, if these covariates exhibit a possibly small but systematic impact on the demographic variable. Hence, in principle, the co-evolution of the demographic variable and its covariates should be modeled together. In our case, this means choosing $N > 0$, resulting in the full model with $z_t = [\kappa_t \quad Y_t]'$ instead of the simpler special case where $z_t = \kappa_t$, according to the Lee–Carter model. The informational gain of this inclusion depends of course on the specifications of the demographic variable and appropriate covariates and has to be weighted against the increased number of parameters to be estimated. By the vector autoregression in Equation (4.2), our model enables the requested utilization of covariates in an appropriate way. Nevertheless, this is only a further alternative to the parsimonious version without covariates, which already exhibits good forecasting features.

4.3.4 Smoothing Along the Age Dimension

When trying to predict future mortality, we have to consider the knowledge about its systematic pattern. To exemplify this point, we might have no idea in the first place about the level of mortality of a 40-year-old 50 years from now; nevertheless, we are very

[6] This issue is discussed extensively in Girosi and King [2008].

confident that this mortality is quite similar to that of a 41-year-old. Hence any forecast missing this basic feature with diverging developments of adjacent age classes should be mistrusted. As already discussed in Section 4.2.2, the Lee–Carter model cannot prevent potential implausible age patterns in out-of-sample forecasts. Our model mitigates this problem. Equation (4.3) guarantees smoothness along the age dimension because the coefficients $\beta_x, \ldots, \beta_{x-q}$ are connected by autoregressive processes. For $\frac{q}{2} \in \mathbb{N}$ and $\alpha_{q/2} \neq 0$, Equation (4.3) can easily be reformulated to get a symmetric representation of smoothing between adjacent age classes:[7]

$$\beta_{\tilde{x}} = \tilde{\alpha}_{-\frac{q}{2}}\,\beta_{\tilde{x}-\frac{q}{2}} + \cdots + \tilde{\alpha}_{-1}\,\beta_{\tilde{x}-1} + \tilde{\alpha}_1\,\beta_{\tilde{x}+1} + \cdots + \tilde{\alpha}_{\frac{q}{2}}\,\beta_{\tilde{x}+\frac{q}{2}} + \tilde{\epsilon}^{\beta}_{\tilde{x}}\,. \tag{4.6}$$

Assuring a plausible age pattern without jumps might be even more important when looking at more volatile data than in our example of current all-cause mortality from the United States, for example, as for the case of single-cause mortality or for data from non-industrialized countries in the past and present.

4.3.5 Cohort Effects

The general model described previously can theoretically be extended to also capture cohort effects. We just have to extend Equation (4.1) with an additional variable corresponding to the cohort dimension, which can be expressed as

$$d_{x,t} \;=\; \overline{d}_x + \beta_x z_t + \beta^{\gamma}_x \gamma_{t-x} + \epsilon^d_{x,t}\,. \tag{4.7}$$

With $N = 0$ Equation (4.7) is similar to the model described in Renshaw and Haberman [2006]. One deviation from their model is that we assume the following law of motion:

$$\gamma_{t-x} = \varphi_1 \gamma_{(t-x)-1} + \varphi_2 \gamma_{(t-x)-2} + \cdots + \varphi_r \gamma_{(t-x)-r} + \epsilon^{\gamma}_t\,, \tag{4.8}$$

where ϵ^{γ}_t is not serially correlated and independent of $\epsilon^d_{x,t}$, ϵ^z_t, and ϵ^{β}_x at all leads and lags. The other deviation to Renshaw and Haberman [2006] is that they estimate Equation (4.7) in a two-step procedure, whereas we would be able to estimate the extended model in a more efficient one-step procedure by introducing an additional step to the Gibbs sampler described in Section 4.6.

[7] Set $\alpha_0 \equiv -1$, $\tilde{\alpha}_i \equiv -\frac{\alpha_{(q/2)-i}}{\alpha_{q/2}}$ for $i \in \{-\frac{q}{2}, \ldots, \frac{q}{2}\}$, $\tilde{x} \equiv x - \frac{q}{2}$, and $\tilde{\epsilon}^{\beta}_{\tilde{x}} \equiv -\frac{\epsilon^{\beta}_x}{\alpha_{q/2}}$.

4.3.6 Indeterminacies

In the estimation procedure we have to deal with three kinds of potential indeterminacies, namely, sign, scale, and rotational indeterminacies. The former two can be illustrated with the following example. Presume we multiply Equation (4.1) by $1 = \frac{\gamma}{\gamma}, \gamma \neq 0$; then we obtain

$$d_{x,t} \ = \ \overline{d}_x + (\beta_x^\kappa \gamma)\left(\frac{\kappa_t}{\gamma}\right) + \beta_x^Y Y_t + \epsilon_{x,t}^d \,. \tag{4.9}$$

Of course, this equation implies the same data-generating process as Equation (4.1), even though we have $\widetilde{\beta}_x^\kappa \equiv \beta_x^\kappa \gamma$ and $\widetilde{\kappa}_t \equiv \kappa_t/\gamma$ with different scale or sign than before. To solve these indeterminacies we need additional constraints. Following the Lee–Carter model, we impose $\sum_{t=0}^T \kappa_t^k = 0$ and $\sum_{x=0}^A \beta_x^k = 1$ for all $k \in \{1, \ldots, K\}$. In the case of $K > 1$ an additional rotational indeterminacy occurs, because appropriate rotations yield

$$d_{x,t} = \overline{d}_x + \left(\beta_x P'\right)\left(P z_t\right) + \epsilon_{x,t}^d \,,$$

where

$$P = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}$$

is an orthogonal matrix with $\widetilde{\widetilde{\beta}}_x \equiv \beta_x P'$ and $\widetilde{\widetilde{z}}_t \equiv P z_t$, implying the same data-generating process as Equation (4.1). Sufficient conditions for unique identification are to set the lower $K \times K$ block of β_x^κ to a diagonal matrix and the lower $K \times N$ block of β_x^Y to zero.[8]

4.4 Predictive Densities

In order to derive analytically distributional statements on the probabilities of outcomes we describe the posterior predictive densities corresponding to the future path of the demographic variables up to horizon H. In this context we find it useful to define

$$d_x^H \equiv \left[d_{x,T+1} \ \ldots \ d_{x,T+H}\right],$$

$$d_x^T \equiv \left[d_{x,1} \ \ldots \ d_{x,T}\right],$$

$$z \equiv \left[z_1 \ z_2 \ \ldots \ z_T\right],$$

$$\beta \equiv \left[\beta_0 \ \beta_1 \ \ldots \ \beta_A\right]',$$

[8]This is similar to the dynamic factor literature. See, among others, Geweke and Zhou [1996] and Bernanke et al. [2005].

$$\Psi \equiv \left\{(c, \phi_1, \phi_2, \ldots, \phi_p, \Sigma_z), (\alpha_1, \alpha_2, \ldots, \alpha_q, \Sigma_\beta), (\sigma_d^2)\right\} .$$

Thus the posterior predictive density can be expressed as

$$p\left(d_x^H | d_x^T\right) = \int\int\int p\left(d_x^H | z, \beta, \Psi, d_x^T\right)\; p\left(z, \beta, \Psi, | d_x^T\right)\; dz\; d\beta\; d\Psi .$$

In order to obtain values for the future path of the observations we draw ϵ^z_{T+i} from $\mathcal{N}(0, \Sigma_z)$ for $i = 1, \ldots, H$ and iterate on

$$z_{T+i} = c + \phi_1 z_{T+i-1} + \phi_2 z_{T+i-2} + \cdots + \phi_p z_{T+i-p} + \epsilon^z_{T+i} . \tag{4.10}$$

Following this, we use the values from Equation (4.10), draw $\epsilon^d_{x,T+i}$ from $\mathcal{N}(0, \sigma_d^2)$, and iterate on

$$d_{x,T+i} \;=\; \overline{d}_x + \beta_x z_{T+i} + \epsilon^d_{x,T+i}$$

to get draws from the joint posterior distribution of d_x^H.

4.5 Priors

We introduce priors on the VAR parameters via dummy observations by simulating an artificial dataset with certain assumed properties and add it to our actual dataset. This goes back to the *mixed estimation* procedure suggested by Theil and Goldberger [1961] and was recently applied by Sims and Zha [1998] and Del Negro and Schorfheide [2004]. We generate dummy observations, implying that the series produced include a random walk process. We do this by centering the probability mass for the first lagged coefficient around 1 and for all subsequent lags around 0, while we subsequently decrease the uncertainty that the coefficients are zero for more distant lags.

We consider the following model:

$$Z^* = X^* \Phi^* + \epsilon^* , \tag{4.11}$$

where

$$Z^* \equiv \begin{bmatrix} \lambda_1 \hat{\sigma} \\ 0_{M(p-1)\times M} \end{bmatrix}$$

and

$$X^* \equiv \begin{bmatrix} \lambda_1\hat{\sigma} & 0 & \cdots & 0 \\ 0 & 2\lambda_1\hat{\sigma} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & p\lambda_1\hat{\sigma} \end{bmatrix},$$

with

$$\hat{\sigma} \equiv \begin{bmatrix} \hat{\sigma}_1 & 0 & \cdots & 0 \\ 0 & \hat{\sigma}_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \hat{\sigma}_M \end{bmatrix},$$

where λ_1 is called the *overall tightness of beliefs* around the random walk prior and $\hat{\sigma}_1, \hat{\sigma}_2, \ldots, \hat{\sigma}_M$ are the empirical standard deviations taken from the first p observations. Increasing values for λ_1 imply that we are more certain concerning our prior and hence the prior gets more weight in comparison to information emerging from the dataset via the likelihood function. Taken values for Σ_z as given, the dummy observations imply the following conjugate prior for our VAR parameters:

$$\Phi^*|\Sigma_z \sim \mathcal{N}\left(vec(\hat{\Phi}^*), \Sigma_z \otimes (X^{*\prime}X^*)^{-1}\right). \tag{4.12}$$

The prior for the AR parameters in Equation (4.3) is similar to the one specified for the VAR parameters with λ_2 as the overall tightness of beliefs of the prior. For the variance of the disturbance in Equation (4.1) we assume an inverted gamma distribution $\mathcal{IG}(\frac{\tau_1}{2}, \frac{\tau_2}{2})$.

4.6 Estimation

We estimate our model using MCMC methods; more precisely, we apply the Gibbs sampler. This method enables us to draw from the joint distribution $\mathcal{P}(\Psi, z, \beta)$ by subdividing it into the conditional distributions $\mathcal{P}(\Psi \mid z, \beta)$, $\mathcal{P}(z \mid \Psi, \beta)$, and $\mathcal{P}(\beta \mid \Psi, z)$ and draw iteratively from them. Taking initialized values for $z^{(0)}$ and $\beta^{(0)}$ as given, we sample in the ith iteration $\Psi^{(i)}$ from $\mathcal{P}(\Psi \mid z^{(i-1)}, \beta^{(i-1)})$, $z^{(i)}$ from $\mathcal{P}(z \mid \Psi^{(i)}, \beta^{(i-1)})$, and $\beta^{(i)}$ from $\mathcal{P}(\beta \mid \Psi^{(i)}, z^{(i)})$ successively. Under weak conditions and for $i \to \infty$ the Gibbs sampler converges and we obtain samples from the desired joint distribution $\mathcal{P}(\Psi, z, \beta)$.[9] For a more detailed description of the estimation procedure see Appendix

[9] See Geman and Geman [1984].

4.

4.7 Data

We apply our model to age-specific total (combining female and male) mortality data from the United States with 91 individual age classes from 0 to 90 as shown in Figure 4.1 as specification of the demographic variable $d_{x,t}$.[10] These time series provided by the Human Mortality Database span the period 1933–2005, of which we use the post-World War II period.[11] We add macroeconomic time series of real GDP per capita and of unemployment, which are displayed in Figure 4.2. The data for real GDP per capita are expressed in logarithms of chained 2000 Dollars, and the unemployment rate is measured as the number of unemployed as a percentage of the civilian labor force.[12]

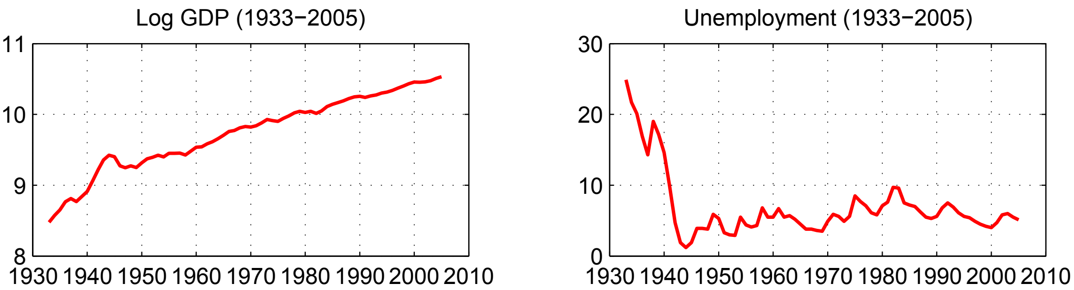

Figure 4.2: Logarithmized GDP and unemployment rate for the United States 1946–2005.

4.8 Results

We apply our model to mortality data from the United States in the period 1946–2005 and gradually vary the model specification. With the objective of comparing with the

[10] Unlike Lee and Carter [1992], where each age class comprises 5 years, we refrain from age grouping and keep the detailed information of single age classes.

[11] See Human Mortality Database [2008]. In the Human Mortality Database raw data are corrected for obvious mistakes and, for the calculation of life tables, death rates for the age classes 80 and above are smoothed by fitting a logistic function according to Thatcher et al. [1998] if the number of observations becomes too small. Wilmoth et al. [2007] supply a detailed method protocol. In the case of the United States, population estimates for 1940–1969 are adjusted to exclude the Armed Forces overseas and to correct for the inclusion of Alaska and Hawaii. Moreover, owing to the lack of data for the age classes 75 and above in the period 1933–1939, the extinct cohort method is applied as supposed by Kannisto [1994].

[12] Although the pre-1947 unemployment figures refer to persons aged 14 and above, whereas the post-1947 figures refer to persons aged 16 and above, this minor change causes no jump in 1947, when both definitions yield the same number. With respect to GDP and the unemployment rate, see the Carter et al. [2006].

Lee–Carter results, we first assume κ_t to consist of only one unobserved time series, which may be called mortality index, and abstain from using covariates. Afterward, the macroeconomic time series are included as covariates.

4.8.1 Preliminaries

For the results we used a lag length of $p = 4$ for the z's and $q = 4$ for the β's. The prior specifications, which we describe in Section 4.5, are $\lambda_1 = 5$ for the VAR parameters of z and a flat prior $\lambda_2 = 0$ for the AR parameters of β.[13] For the variance of the disturbances in Equation (4.1) we choose $\tau_1 = 0.01$ and $\tau_2 = 3$.

The estimation results may be affected by the choice of time period and age span under consideration. To check whether our results depend on the initial β parameters we conduct the following exercise. We leave out mortality of the youngest age classes and estimate our model with $\beta_s, \ldots, \beta_A$, where $s > 0$. We obtain very similar results to the full model $\beta_s, \ldots, \beta_A$, suggesting that the choice of initial values for the β's does not bias our results. With respect to the time period we mainly focus on the postwar era 1946–2005 to base the analysis and forecasts on circumstances relatively close to the present and to avoid the influence of very high unemployment after the Great Depression and possible distortions from World War II. Nevertheless, we also test for specifications that span the entire period 1933–2004 and get very similar results for the forecasts.

To ensure that our Gibbs sampler converges we restart the algorithm several times, each time using different starting values drawn from an overdispersed distribution. The results for all these different chains are very similar. Our sampler already reaches convergence after a few thousand draws. Furthermore, to keep the starting values from influencing our results we discard the first half of the chain as the burn-in phase.

4.8.2 One Kappa but No Covariates

First we present the simplest version, with only one latent variable κ and no covariates. Figure 4.3 shows the estimated κ and the corresponding coefficient matrix β, which reveals how close the mortality of particular age classes is associated with development of the latent variable κ. The age classes 0–15 are higher than average exposed to κ; however, all age classes are positively related to the latent variable.

[13] $\lambda_1 = 5$ is also used by, among others, Sims and Zha [1998].

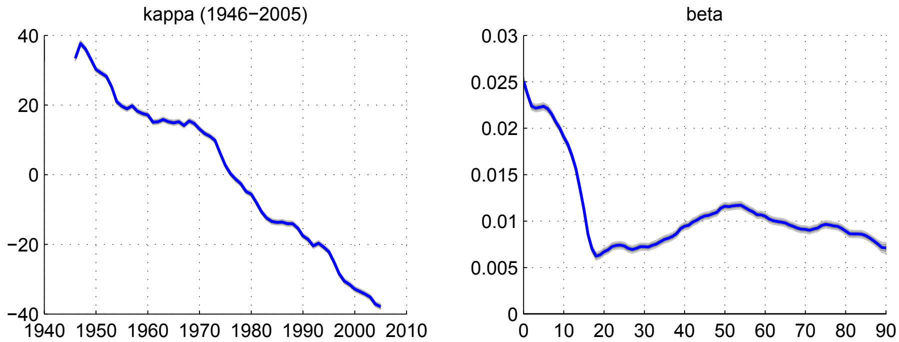

Figure 4.3: Estimated κ and β. The small gray shaded area around the blue median represents 90% of the posterior probability mass regarding both parameter and random term uncertainty.

In Figure 4.4 we show different in-sample forecasts for κ over a 15-year horizon from 1991 onward, that can be compared with the "realized" developing (red line), which means the median of the estimated κ for the entire period.

Additionally, we show in Figure 4.5 out-of-sample forecasts for a longer horizon up to the year 2050. These forecasts are of course subject to different kinds of uncertainty. In each case, we give an overview of forecasts, where either only the uncertainty due to the random terms ϵ, only the uncertainty due to the estimation of the parameters of the model, or both kinds of uncertainty are considered. The resulting distributional features of the forecasts are illustrated by the probability mass around the medium forecast. In all cases, accounting only for the random term uncertainty results in quite close forecasts which have the form of a parabola and widen only a little over time. In contrast to this, the forecasts accounting only for parameter uncertainty start very narrow but widen faster than they do linearly. The forecasts with respect to both sources of uncertainty are of course the widest. In this case, the overall accuracy of the forecast is dominated by the effect of the random term in the short run and by the effect of the parameter estimation in the long run.[14] This result demonstrates the extent to which presentations of forecasts can be misleading by giving rise to an illusion of sureness if important sources of uncertainty are ignored. Moreover, even the most precautious versions of our plots give only lower bounds for the real forecast uncertainty, which can be even larger, because the specification of the model (model choice) and the estimation of κ in the observation period (starting point for the forecast) are also nondeterministic.

[14]Lee and Carter [1992] mention a dissenting relationship in their Appendix B.

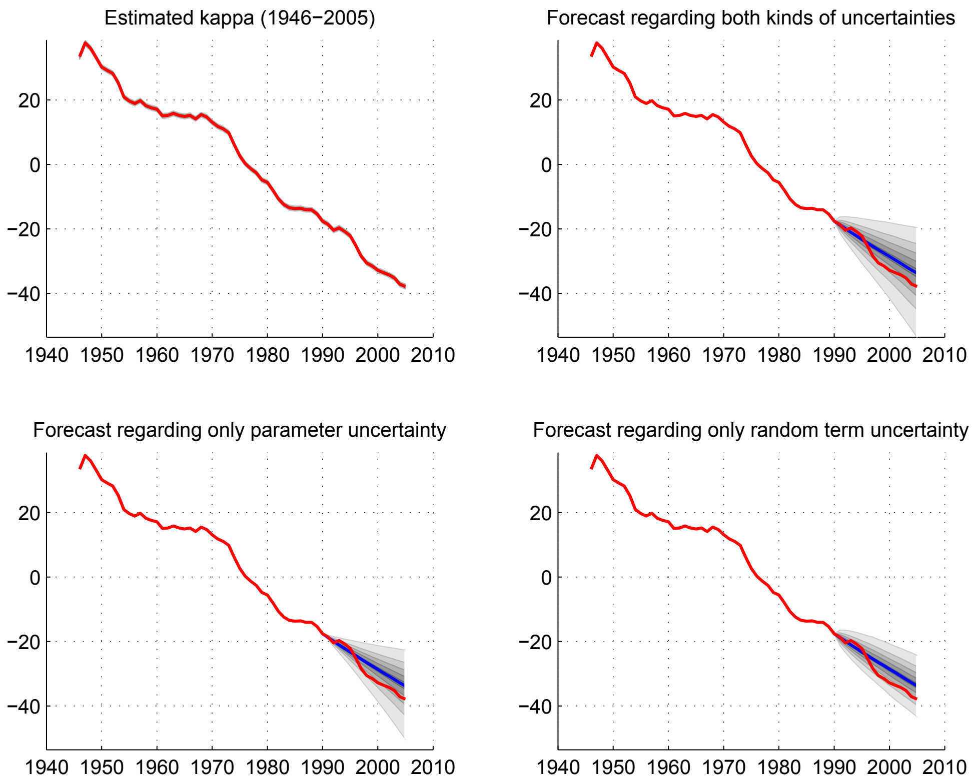

Figure 4.4: Panel with in-sample forecasts of κ with respect to different sources of uncertainty for the period 1991–2005. The red line always displays the median estimation of κ based on the observations for the whole period 1946–2005. The blue line displays the median forecast of κ based only on the information up to 1990. The entire gray shaded area represents 90% of the posterior probability mass and each of the different gray shaded bands represents 10% of the posterior probability mass. Note that the innermost band is largely covered by the blue line.

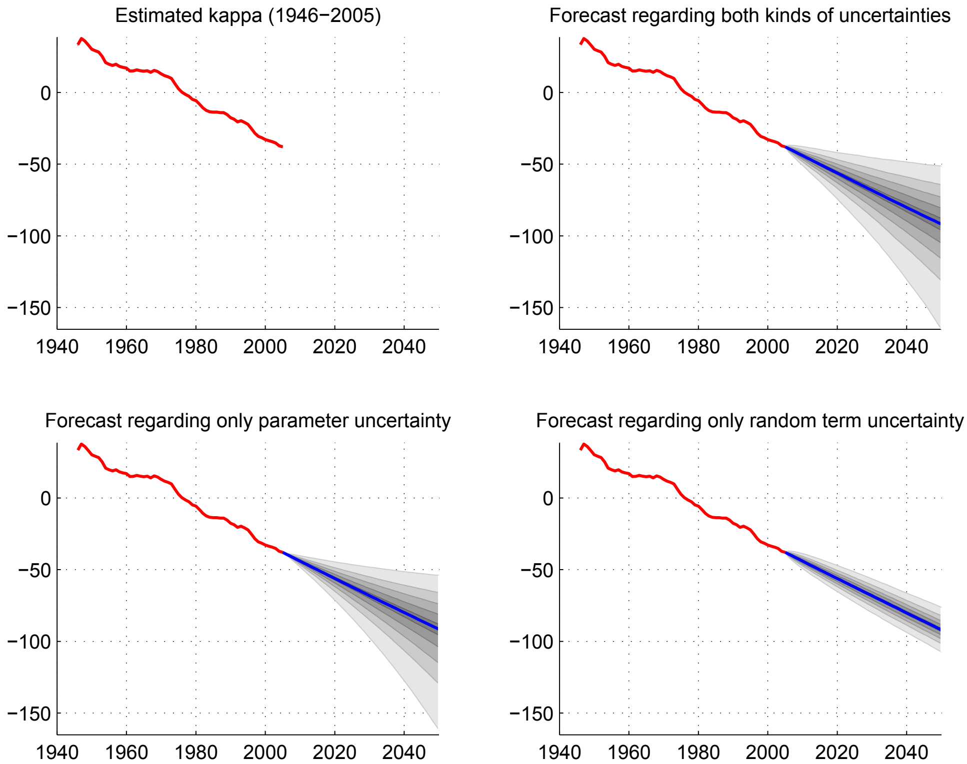

Figure 4.5: Panel with long-run forecasts of κ with respect to different sources of uncertainty for the period 2006–2050. The red line displays the median estimation of κ based on the observations in the period 1946–2005. The blue line displays the median forecast of κ based on this information. The entire gray shaded area represents 90% of the posterior probability mass and each of the different gray shaded bands represents 10% of the posterior probability mass. Note that the innermost band is largely covered by the blue line.

4.8.3 Covariates and Additional Kappa

In order to improve our predictions we extend our model by including logarithmized real GDP per capita and the unemployment rate as covariates and, in a further step, by adding a second latent variable κ_2 to the specification with the two covariates. Figure 4.6 shows the estimated coefficients β related to κ_1 and κ_2, GDP, and unemployment, revealing the extent to which age-specific mortality is affected by the latent variables and covariates. Of course, this paves the way for structural analysis of the systematic interactions of mortality and covariates using impulse responses analyses, which is presented in detail in Reichmuth and Sarferaz [2008a].

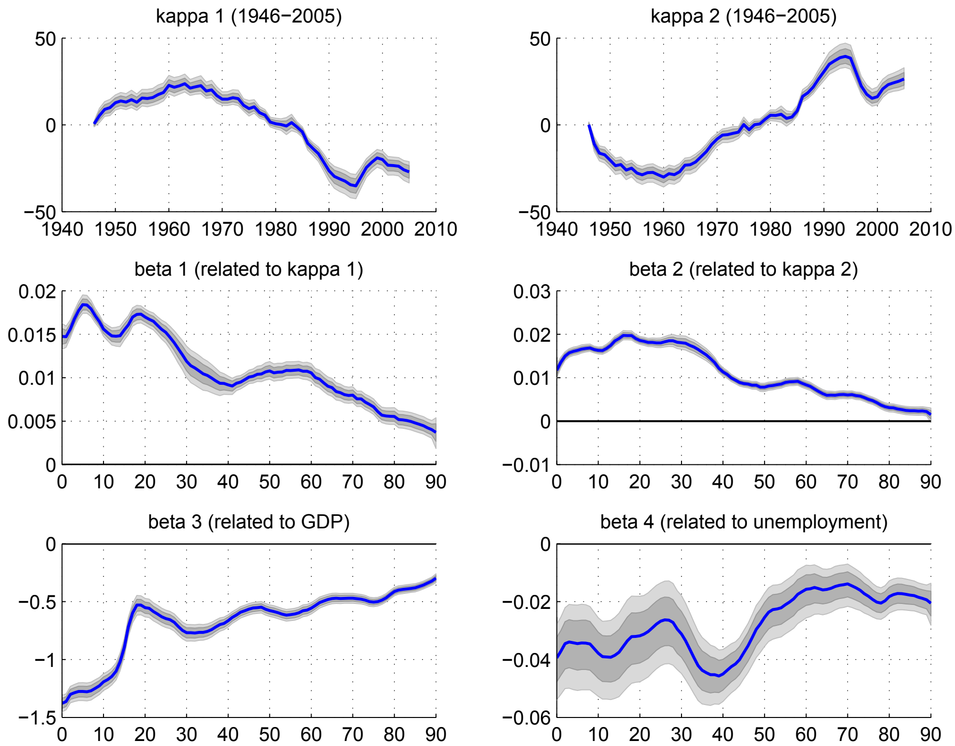

Figure 4.6: Estimated κ's and β's for the model specification with two latent variables and GDP and unemployment as covariates. The entire gray shaded area around the blue median represents 90% of the posterior probability mass and the dark gray shaded area represents 68% of the posterior probability mass regarding both parameter and random term uncertainty.

For the simplest specification without covariates, Figure 4.7 shows the median out-of-sample forecasts of age-specific mortality about the middle and at the end of the forecast period in comparison to actual observations. As can be seen, the overall level of mortality declines steadily but the shape stays more or less the same. Figure 4.8 shows different

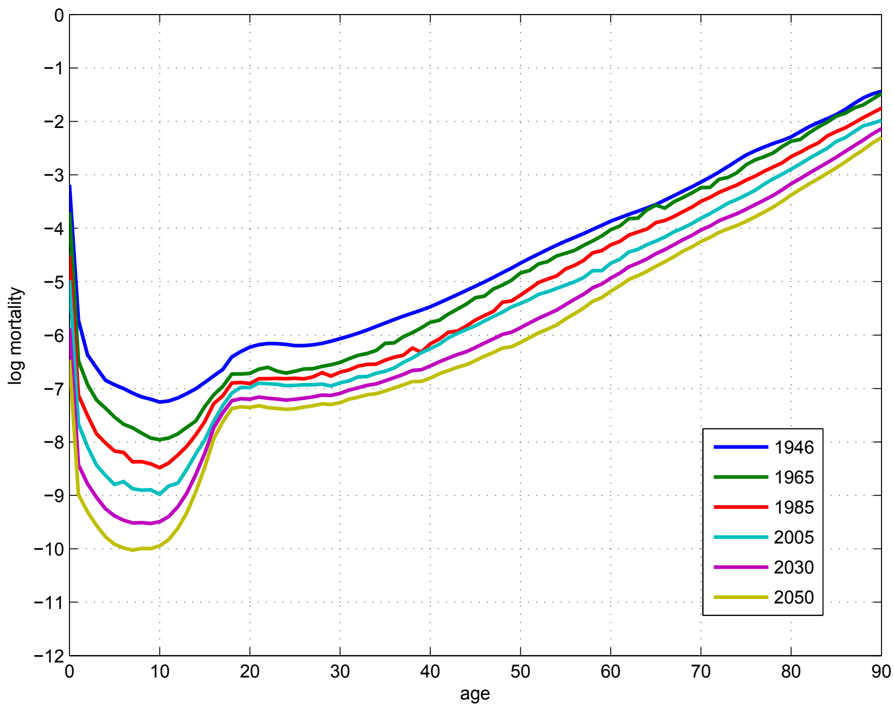

Figure 4.7: Observations and forecasts of age-specific mortality $m_{x,t}$ at different points in time. The lines for the years 2030 and 2050 display the median forecasts regarding both parameter and random term uncertainty.

out-of-sample forecasts for the longer horizon until 2050, where the error bands widen by time. As can be seen in the first and second rows of Figure 4.8, including macro variables as covariates improves the forecasts for the higher age classes, whereas the forecasts for the age classes 15–40 deteriorate. This leads us to the conclusion that covariates have to be chosen very carefully in general, as they might help predict particular age classes but at the same time worsen the forecasts of others. The third row of Figure 4.8 shows that adding κ_2 to the specification with two covariates improves the forecasts again. For the age classes below 15 or above 40, they are the best of all specifications.

The figures discussed in this section demonstrate the smooth transition along the age dimension as described in Section 4.3.4. Admittedly, the difference to the Lee–Carter results is not so obvious owing to their previous age grouping, but note that we prevent divergence for single age classes in the long-run independent of the choice of all-cause mortality.

The forecast errors presented in this chapter can be interpreted differently, depending on the particular research interest of the reader. For example, overestimating future mortality may jeopardize pension schemes, whereas underestimating is a danger for life insurance calculations. In both cases major misjudgments have more severe consequences for the stakeholders than smaller ones. This means that not only mean and variance but also higher moments (skewness and kurtosis) of the distribution of predicted mortality matter for the risk assessment. Our Bayesian presentation of the forecast results with a detailed allocation of probability masses provides the information needed.

Moreover, the relatively wide dispersion of our forecasts assigns only a rather low probability for realizations close to the median, which further challenges traditional forecast methods with misleadingly tight error bands.

4.8.4 Life Tables

Life tables deliver some intuitively interpretable variables such as surviving probabilities and life expectancies, which can be calculated from a complete set of age-specific mortalities. For this purpose, we use the simplest specification of our model with one latent variable κ and no covariates to forecast mortality for all age classes up to 110+.[15] We do so for female and male mortality separately, because the resulting life tables are quite different and would not be represented adequately by a version for "total" mortality. Finally, we compute the respective period life tables up to the year 2050 and present the results for females. The detailed calculations are given in Appendix 5. Note that the life table variables depend nonlinearly on a whole set of mortalities at different ages. Thus, to get proper percentiles for the forecasts of these variables, we do not use percentiles of age-specific mortality directly but compute the life tables from the particular mortalities for the second half of 30,000 independent draws separately. Once again, the error bands with respect to both parameter and error term uncertainty are the widest.

Figure 4.9 displays the hypothetical birth-time probabilities $l_{x,t}$ of surviving up to the exact age x if a female were subject to the age-specific mortalities of one particular period over her whole life cycle. During the observation period 1946–2005 the curves consistently move upward and to the right. First, reductions of child mortality mainly shift the curve upward, whereas later on reductions of old-age mortality shift it to the right. The forecast for 2050 shows that this trend will probably continue, though the

[15] The inclusion of very high ages is necessary for the best possible calculation of remaining life expectancies.

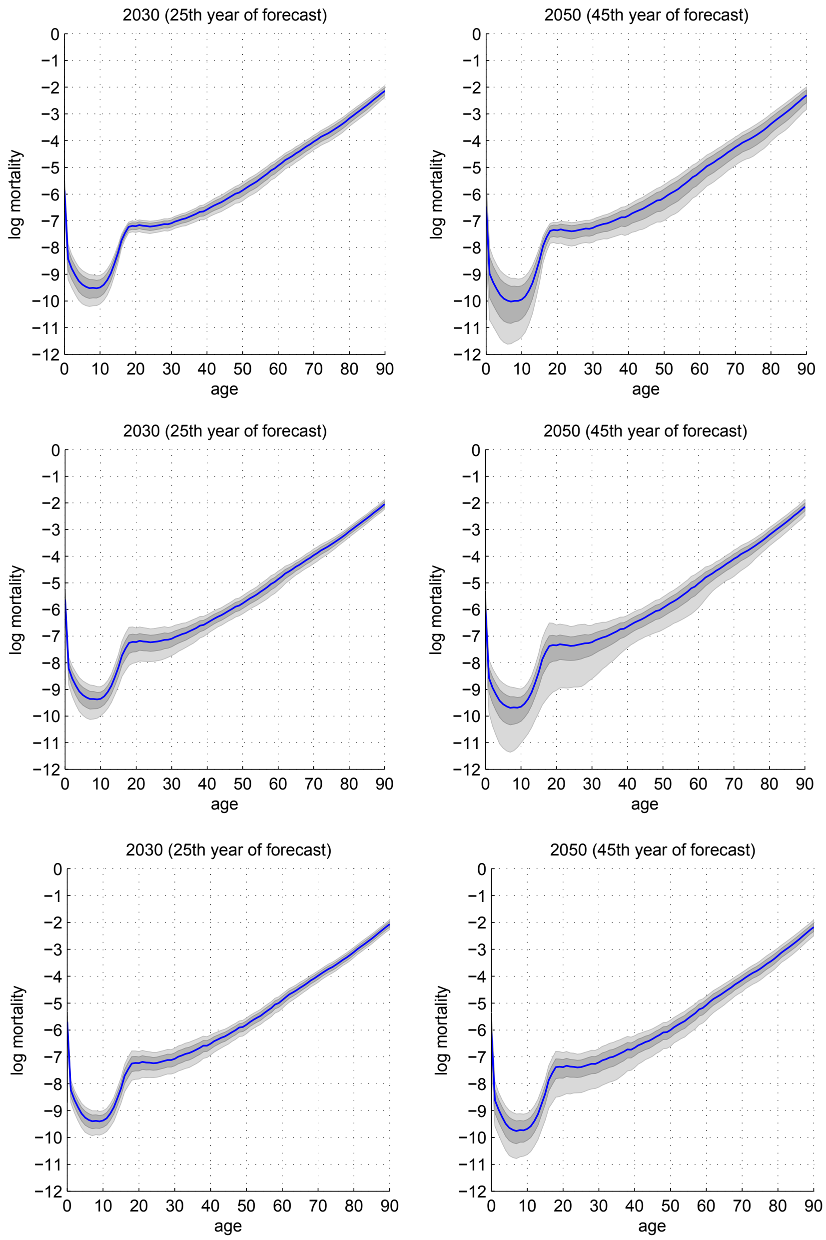

Figure 4.8: Panel with forecasts of age-specific mortality $m_{x,t}$ 25 and 45 years ahead for different model specifications. The first row shows the specification for $K = 1$, $N = 0$, the second row for $K = 1$, $N = 2$, and the third row for $K = 2$, $N = 2$. The entire gray shaded area around the blue median represents 90% of the posterior probability mass and the dark gray shaded area represents 68% of the posterior probability mass regarding both parameter and random term uncertainty.

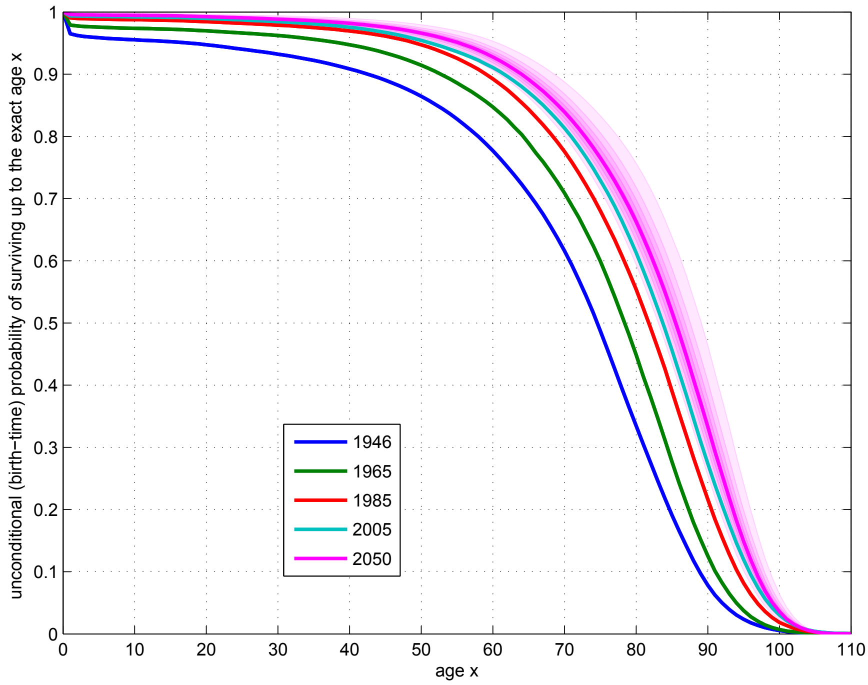

Figure 4.9: Probabilities $l_{x,t}$ of surviving up to the exact age x for females based on period life tables for different points in time. The figures for the years 1946–2005 are calculated from observations. The thick magenta line displays the median forecast of $l_{x,2050}$. The entire magenta shaded area represents 90% of the posterior probability mass and each of the different magenta shaded bands represents 10% of the posterior probability mass regarding both parameter and random term uncertainty. Note that the innermost band is largely covered by the thick line for the median.

error bands show the relatively high uncertainty about the future survival curve. However, the forecast accuracy of the life table variables, which depend in particular on old-age mortality, can also be improved by the inclusion of covariates.

Figure 4.10 displays the corresponding birth-time probabilities $d_{x,t}$ of dying at age x. Of course, the values rise over most of the lifetime and peak somewhere in old age before they fall again.[16] Remarkably, these probabilities not only shift to the right but also concentrate increasingly on a smaller age range. With respect to the survival curve, this corresponds to a transformation toward a long relatively flat initial course followed by a steep fall, which is known as rectangularization.

Finally, in Figure 4.11 we present time series of life expectancies at different ages for the

[16] In today's industrialized countries child mortality is no longer a major threat.

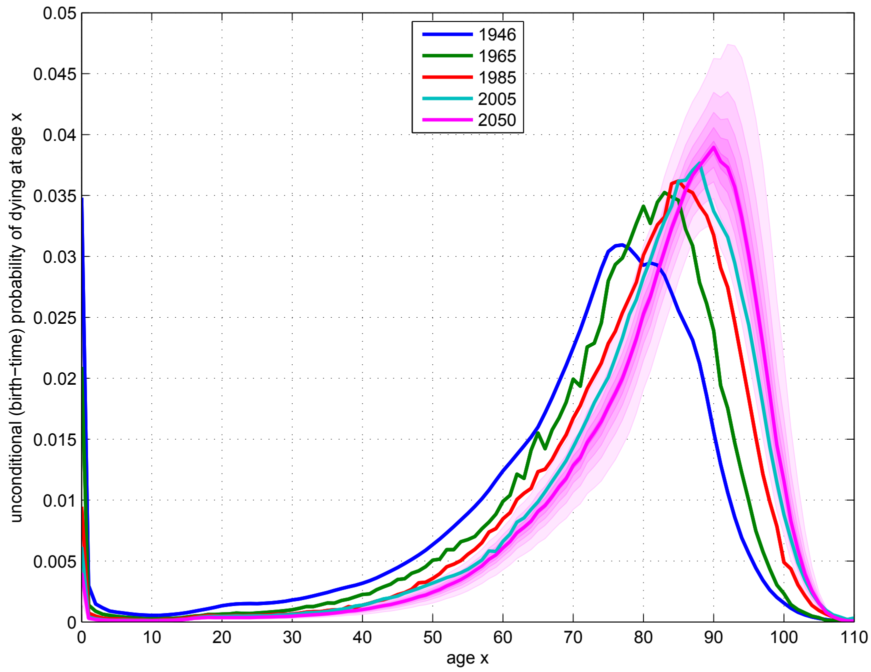

Figure 4.10: Probabilities $d_{x,t}$ of dying at age x for females based on period life tables for different points in time. The figures for the years 1946–2005 are calculated from observations. The thick magenta line displays the median forecast of $d_{x,2050}$. The entire magenta shaded area represents 90% of the posterior probability mass and each of the different magenta shaded bands represents 10% of the posterior probability mass regarding both parameter and random term uncertainty. Note that the innermost band is largely covered by the thick line for the median.

whole observation plus the forecast period 1946–2050. Life expectancy always means the remaining life expectancy for those who have already achieved a particular age. In our application, the life expectancies of older people are always lower than those of younger people, because there is no phase of life with such a high mortality that survivors of this phase would have a higher remaining life expectancy than younger people prior to this phase. The life expectancies for all age classes increase quite evenly over time. The rise for the younger people is the strongest, because they benefit from the mortality reduction at all age classes lying ahead of them. Our forecasts clearly show that the trend of increasing life expectancies at all age classes will continue with high probability. For example, the median forecast of the gain in female life expectancy based on period life tables between 2005 and 2050 is about 4.5 years for a newborn and 2.8 years for a 60-year-old. Once again, the error bands of the forecasts can be further reduced by including covariates.

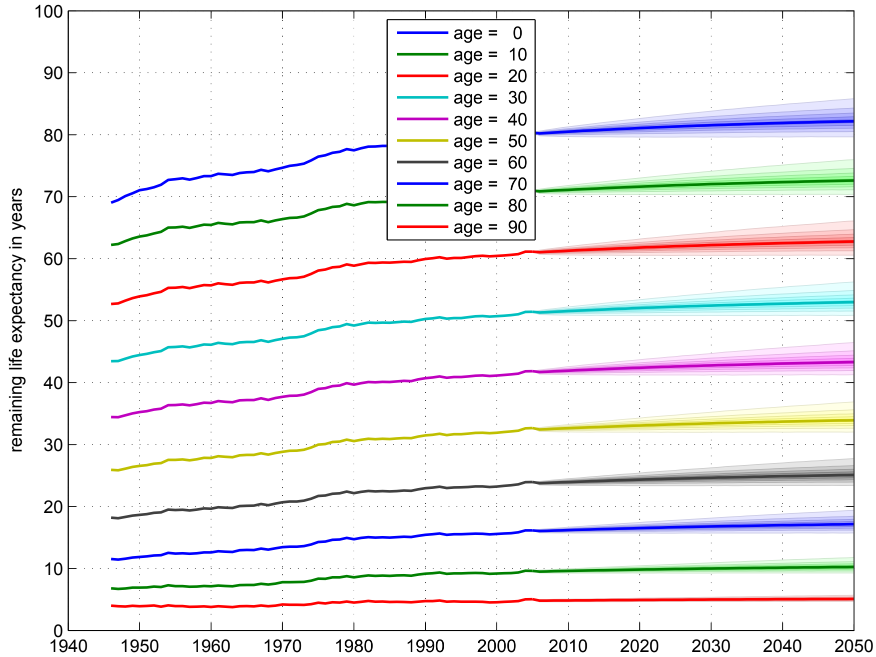

Figure 4.11: Remaining life expectancies $e_{x,t}$ for females of different age classes based on period life tables. The thick lines display figures calculated from observations in the period 1946–2005 and median forecasts for $e_{x,t}$ in the period 2006–2050. For each age class the entire shaded area represents 90% of the posterior probability mass and the different shaded bands represent 10% of the posterior probability mass regarding both parameter and random term uncertainty. Note that some of the bands are largely covered by the thick lines for the medians.

4.9 Conclusion

In this chapter we present an alternative approach to modeling age-specific mortality. We build on the model introduced in Lee and Carter [1992] and extend it in several dimensions. We incorporate covariates and model their dynamics jointly with the latent variable underlying mortality of all age classes by a VAR process. Furthermore, we resolve the shortcomings in the age dimension from which previous models suffered by connecting adjacent age groups through an AR process. Our new modeling approach thus allows for consistent forecasts of age-specific mortality and the other variables.

We develop an appropriate MCMC algorithm, which enables us to estimate the parameters and latent variables jointly in an efficient one-step procedure. With our Bayesian approach we formalize priors for the parameters and thus include information into our model in a formal way. Additionally, we are able to assess uncertainty intuitively by constructing error bands for our forecasts.

We apply our model to U.S. mortality for 1946–2005 and test its forecast ability by means of in-sample and out-of-sample forecasts up to the year 2050. Our model performs well, that is, the forecasts exhibit smoothness along the age dimension with sufficiently tight error bands. Comparing different specifications, it turns out that covariates can indeed help improve the forecasts for particular age classes. Moreover, we demonstrate that uncertainty stemming from the error term is more important in the short run, whereas parameter uncertainty is very important for long-run forecasts. This points to the danger that existing forecasting methods for age-specific mortality, which ignore certain sources of uncertainty, may yield misleadingly sure predictions.

5 The Influence of the Business Cycle on Mortality

We analyze the impact of short-run economic fluctuations on age-specific mortality using Bayesian time series econometrics and contribute to the debate on the procyclicality of mortality. For the first time, we examine the differing consequences of economic changes for all individual age classes. We employ a recently developed model to set up structural vector autoregressions of a latent mortality variable and of unemployment and GDP growth as main business cycle indicators. We find that young adults noticeably differ from the rest of the population. They exhibit increased mortality in a recession, whereas most of the other age classes between childhood and old age react with lower mortality to increased unemployment or decreased GDP growth. In order to avoid that opposed effects may cancel each other, our findings suggest to differentiate closely between particular age classes, especially in the age range of young adults. The results for the United States in the period 1956–2004 are confirmed by an international comparison with France and Japan. Long-term changes in the relationship between macroeconomic conditions and mortality are investigated with data since 1933.

5.1 Introduction

Most people are happy and willing to work hard, when they find a new job during an economic upturn after a phase of economic hardship. Commentators emphasize the improved situation and express the hope for a long duration. In the face of the common satisfaction, is it possible that there is also a hidden dark side of the boom with an increased risk of death? If such a relationship between mortality and the business cycle exists, it potentially affects the entire population, not only the working. Everyone might be subject to unexpected severe implications of economic fluctuations without even knowing about.

The principle concern for the interactions of economy and mortality is certainly not new. In the historical context of a society living at the subsistence level, a Malthusian relationship between mortality and the economy is often assumed, in which shrinking real wages caused by population growth or a crop failure inevitably increased mortality.[1] However, this chapter is not about history, but rather about the United States and other industrialized countries in recent decades. There is a broad literature relating mortality to the state of the economy in modern times, which is discussed in detail later on.[2] In particular, e.g. Brenner [1971, 1979] claimed that the traditional connection of low mortality in good and high mortality in bad economic times is still effective. This conventional socio-epidemiological wisdom of countercyclical mortality has recently been reversed by Ruhm [2000], who found evidence for procyclicality.

In general, the literature neglects age-specific differences in mortality fluctuations. Moreover, in most cases unemployment is the only business cycle indicator. We extend the analysis by an adequate inclusion of the age dimension of mortality and discriminate between the impact of changes in unemployment and in the growth of the gross domestic product (GDP). In the main, we analyze mortality changes along the business cycle in the United States since 1956 on an annual basis. In addition, we include data since 1933 covering the long aftermath of the Great Depression and World War II. Identical analyses for France and Japan enable us to draw an international comparison of the particular relations between mortality and the business cycle in countries with different economic and demographic experiences and very different institutional settings. We always take a macro perspective and do not use micro data on individual life courses, that is, we do not claim that exactly the same persons are hit by unemployment and changed mortality, but the society as a whole.

Of course, only some people lose their job in a recession. Nevertheless, many more may be affected indirectly if unemployment is correlated with changing working conditions or working habits of the still employed. Diffusion of lifestyle changes via social networks forms another possible channel, how job losses of some can alter health and mortality of many. As well as other aspects of human behavior, the health related risky or preventive behavior of individuals is influenced by the example of others. Smoking, alcohol abuse, excessive calories intake, and deficient physical activity are obvious examples of risky behavior changing collectively over time. The reason is not only that people share many activities in their peer group. Social norms determine, what is widely considered as usual or at least acceptable in a community and what is not. For exam-

[1] The idea goes back to Malthus [1798].

[2] See Section 5.2.

ple, Christakis and Fowler [2007] show that the risk of obesity is connected with the prevalence of obesity in the peer group of an individual, but also transmitted in the social network between distant people, who have no direct contact at all. This results in *social infections* or *social epidemics*. Thus, life style changes of some people, who are especially affected by economic shocks, may have an influence on the many others not affected directly, and general mortality may change in line with general economic conditions.

An objection against economically induced mortality fluctuations might be that the wave length of the business cycle is short compared not only with the entire human life cycle, but also with the courses of many lethal diseases. At a first glance, this might indeed challenge any major relation between mortality and the business cycle. Nevertheless, some reasoning about actual causes of deaths refutes this argument. It is self-evident that the incidence of deaths from external causes like infections, accidents, or violence may change in principle at least as rapidly as the state of the economy does. Even if the incidence of these fatalities is quite low, they still may account for a significant variation in mortality. For many other causes of deaths it might seem less plausible to assume short-run fluctuations. For example, myocardial infarction as a major preventable cause of death is typically associated with a long course of gradual deterioration of the health status, often related to decades of unhealthy behavior. However, we have to distinguish between long-run causes of a precarious health status and very short-run incidents which finally trigger the infarction itself. The importance of short-run circumstances is demonstrated by significant variations of the incidence of myocardial infarctions depending on the day of the week.[3] Altogether, there is strong evidence that mortality from preventable diseases, which accounts for a big portion of all fatalities, is subject to short-run influences as well.

So far, we have argued that the analysis of short-run interactions of mortality and the economy, both considered in a macro perspective, is reasonable. In this chapter, we apply a Bayesian time series approach to demographic modeling, which is presented in detail in Chapter 4. The model specified therein is characterized by an appropriate embodiment of age-specific characteristics and can be estimated by a sophisticated Markov

[3]For example, Willich et al. [1994] and Spielberg et al. [1996] find these variations in both West and East Germany prior to reunification. Anson and Anson [2000] find weekly cycles of mortality from external as well as from internal causes like circulatory diseases in Israel. As a further example, Sargent et al. [2004] analyze the effect of a local smoking ban in Montana and find that the incidence of infarctions is significantly reduced already in the first six months after a prohibition of public smoking.

chain Monte Carlo (MCMC) algorithm. A latent mortality variable, which drives the common part of the development of age-specific mortalities, and the two business cycle indicators as covariates are jointly modeled in a vector autoregression (VAR). Complemental autoregressions prevent sudden changes of the coefficients, which link these variables with particular age-specific mortalities. The VAR model accounts for potential endogeneity of all variables. The estimated VAR parameters allow for a structural analysis of the actual interrelations of the latent mortality variable and the different covariates by means of impulse response functions. Based on these mutual interactions and the respective coefficients linking the variables to age-specific mortality, we calculate the full pattern of age-specific reactions to shocks in the economic variables. We trace both the reactions at a fixed age and the reactions of a real cohort aging by time. The first choice corresponds to a cross section and the second one to a diagonal section in the three dimensional surface of mortality reactions. In all cases, the Bayesian estimation approach yields not only point estimates, but information on the whole distribution of the results. Thus, we present error bands with probability masses corresponding to different percentiles of the responses of age-specific mortality.

We contribute to the debate on the effects of the business cycle on mortality triggered by recent findings of procyclical mortality contradicting conventional socio-epidemiological wisdom. For the United States in the period 1956–2004 we find that the reaction of the male twenty to thirty years olds to macroeconomic shocks constitutes an exception. While most other age classes react negatively to a shock in unemployment and positively to a shock in GDP growth, the 25 years olds react with reversed signs. We confirm these findings with international data from France and Japan and observe that in both countries, in addition to the young male adults, even the 30 to 40 years olds react differently. This suggests that when examining the relationship between business cycles and mortality, data including all single age classes should be used in order to avoid spurious results. To investigate a change in the relationship between macroeconomic variables and mortality rates we use data for the period 1933–1969. We find that the relationship has indeed evolved over time. In the earlier period, all age classes react procyclically in the short-term and mostly countercyclically in the mid-term.

The rest of the chapter is organized as follows: Section 5.2 provides a summary of relevant literature on mortality and the business cycle. In Section 5.3 the model is stated and Section 5.4 briefly addresses the estimation procedure. The data are described in section 5.5. The empirical results are presented in Section 5.6. Finally, Section 5.7

presents our conclusions.

5.2 Literature on Mortality and the Business Cycle

In this section, we review the main literature on interactions of economic conditions and mortality in the modern world.[4] Evidence for a negative correlation in the long-run and in cross-section studies is compiled with a number of findings of procyclical mortality with respect to the business cycle. A short digression on the possible long-run implications of economic conditions completes the section.

5.2.1 Negative Correlation of Mortality and Good Economic State

Due to the impressive decline of mortality accompanied with an enormous rise of economic output and individual wealth since the 19th century, the correlation between mortality and good economic conditions is clearly negative in the long run.[5]

The same negative correlation is found in cross-sections of countries with different development levels, for example, in the World Mortality Report,[6] or of individuals with different socio-economic positions within one country. This link between mortality and economic well-being turns out to be superior to many other attempts to explain differential mortality. For example, Menchik [1993] finds monotonic negative influence of the economic status on mortality in the United States and largely disproves genotype differences of ethnic groups as reason for mortality differences. Deaton [2003] discards inequality of income as a major determinant of mortality, but underscores the importance of income level as health determinant.[7] von Gaudecker and Scholz [2007] also find a large, monotonic impact of lifetime earnings on mortality in Germany and disprove at the same time sustained effects of having lived under the opposed institutional environments of either West or East Germany prior to reunification. Mackenbach et al. [2003] point out that in previous studies for all countries with available data, mortality turns out to be higher for those with low socio-economic position, no matter whether this is indicated by educational achievement, occupational class, or income level.

Nevertheless, these unchallenged and well-known facts are about correlation, but not necessarily about causation. Furthermore, neither the look at long-run developments

[4] We do not consider the literature on pre-industrialized societies of the Malthusian era.
[5] For the rise of life expectancy owing to fallen mortality, see, for example, Oeppen and Vaupel [2002].
[6] See United Nations [2006].
[7] This finding is only about income inequality, not about different ranks in the social environment.

nor a simple comparison of different objects of investigation without adequate time perspective can reveal effects of short-run fluctuations. In the context of the business cycle, this means that conflicting settings in which mortality is either countercyclical, acyclical, or even procyclical would all be perfectly compatible with these facts as long as mortality follows an downward time trend, whereas economic output follows an upward time trend beside their particular short-run fluctuations, that is, time acts as a confounding factor.

Although a positive correlation of mortality and the business cycle was already discovered by Ogburn and Thomas [1922] for the United States 1900–1920 and six states of the U.S. 1870–1920, this result was widely ignored later on.[8] Instead, the most influential work on the interrelation of changes of economic conditions and mortality is done by e.g. Brenner [1971, 1979], who finds countercyclical mortality in aggregated time series. However, his studies are challenged by a number of authors for methodological reasons.[9]

5.2.2 Recent Findings of Procyclical Mortality

More recently, several panel studies deliver strong evidence that mortality is procyclical with respect to the business cycle. In most cases, the unemployment rate is used as economic variable. Since unemployment is countercyclical with respect to the business cycle, mortality changes are called procyclical if they are reverse to changes in unemployment as explanatory variable.

The most prominent results are due to Ruhm [2000], who applies a fixed-effect model to state level data for 1972–1991 from the United States and finds that a sustained increase of unemployment instantaneously leads to significantly decreased mortality persisting for several years. Among adults, mainly the younger age class of 20–44 year olds is affected and much less the older classes, in which most of the fatalities occur. Among ten major causes of death only suicides increase and fatalities from cancer stay unchanged, whereas all other types of mortality decrease. The decrease is particularly strong for vehicle accidents and homicide, but also not negligible for other accidents, heart dis-

[8] The authors themselves were skeptical about their findings. Decades later, similar results are found. Eyer [1977] already addresses the different causes of death and their changes over the business cycle, while Higgs [1979] looks at data from large U.S. cities 1871–1900 and relates mortality to procyclical immigration.

[9] Ruhm [2000] gives an overview.

eases, and infant mortality.[10] In general, the more strongly fluctuating causes of deaths predominantly strike younger people.[11] A supplementary analysis of micro data for 1987–1995 from the annual Behavioral Risk Factor Surveillance System (BRFSS) survey yields that reported lifestyle changes are consistent with the resulting changes of mortality.[12] Ruhm [2003] complements with the analogous finding that morbidity is procyclical, too, in particular with regard to acute health problems and to people in the prime-working age.

The findings of Ruhm [2000] are confirmed by Neumayer [2004], who uses state-level data of unemployment from Germany in the period 1980–2000. He obtains some differing results concerning the individual causes of deaths, but also finds procyclical general mortality, in particular for the age classes below 45 and above 65.[13] A study on the country-level with data from 23 OECD countries in 1960–1997 by Gerdtham and Ruhm [2006] also confirms the procyclical effect of changes of unemployment on mortality.

In contrast to these panel studies, Tapia Granados [2005] applies a time series approach.[14] The change rates in age-adjusted general mortality as well as the change rates in specific mortality for population sub-groups, different age classes, and particular causes of death are regressed on the change rates in unemployment and GDP in the United States for 1900–1996. Independent of the choice of the indicator for the business cycle, mortality is found to be procyclical for the entire period as well for all examined sub-periods.[15] He estimates that a time trend of falling mortality more than

[10] The decomposition of predicted cyclical fluctuations of mortality yields that due to their high elasticity, car accidents and other external causes (such as other accidents, suicide, and homicide) account for a high percentage of the variation in spite of their relatively low incidence of fatalities. Deaths from cancer form the other extreme with high incidence, but low fluctuations, whereas so-called preventable deaths (from heart or liver diseases as well as from infectious diseases) exhibit the highest weight in both variation and incidence.

[11] With respect to deaths from coronary heart disease as the biggest single cause of death, Ruhm [2006] finds working-age and older people to be affected similarly. These fatalities are procyclical and, most notably, react faster than other disease-related causes of death to the business cycle.

[12] Ruhm and Black [2002] and Ruhm [2005] also analyze behavioral changes over the business cycle. They regress individual data from the BRFSS (1987–1999/2000) on state-level unemployment and find that tobacco and alcohol consumption as well as obesity are procyclical, whereas leisure-time physical activity is countercyclical. All changes in the prevalences are concentrated among people with worrying health related behavior, who are either heavy users, severe obese, or completely physical inactive.

[13] His main results are robust to replacing the unemployment rate by GDP growth as economic indicator.

[14] In an earlier time series study, Graham et al. [1992] find a countercyclical effect of consumption expenditures and a procyclical effect of unemployment on age-adjusted mortality in the United States in 1950–1988. They also report the cyclical patterns of major causes of death.

[15] In an even longer perspective on Sweden, Tapia Granados and l. Ionides [2008] find that a negative effect of economic growth on mortality in the first half of the 19th century ultimately turns into a positive one in the second half of the 20th century.

offsets the mortality raising effect of growing GDP.[16] In a similar approach, Hanewald [2008] regresses changes of mortality in (West) Germany in 1956–2004 on changes in a multitude of economic, environmental, and behavioral time series. With respect to the economic indicators, she finds an instantaneous positive effect of GDP on mortality, which is clearly more important than the negative effect of unemployment.[17] Unlike Ruhm [2000] and Neumayer [2004], who distinguish only three crude age classes 20–44, 45–64 and 65+, Tapia Granados [2005] uses seven age classes from 0 to 84 and Hanewald [2008] uses eight age classes between 25 and 99. The regressions for the particular age classes are always independent from each other by construction.

In contrast to the results for the OECD, Bhalotra [2007] and Baird et al. [2007] find counter-cyclical infant mortality in India and the developing world. The effect of economic circumstances on old-age mortality is analyzed by Snyder and Evans [2006]. They exploit the quasi natural experiment of a cut in social security in the United States and find that those born after the key date, who get lower payments and do at the same time more post-retirement work, have lower mortality than those born before.

5.2.3 Long-Run Impact of Economic Conditions on Mortality

Adult mortality can even be affected by economic conditions much earlier in the life cycle. Barker [1992] advocates effects of fetal malnutrition on adult health via epigenetic programming of the unborn for the best fit to the current state of the world.[18] Some natural experiments in history deliver evidence for drastic long-run effects. For example, van den Berg et al. [2007] find higher mortality among older men, who were exposed to the Dutch Famine 1846–1847 in their perinatal period, and Almond et al. [2007] find negative effects of the Chinese Famine 1959–1961 on a whole bunch of socio-economic characteristics later on in life. These findings support the point that in addition to age and time their combination as cohort effect is sometimes relevant for mortality, too. Actually, long-run effects on mortality via fetal and neonatal programming are not limited to such rare extreme events. For example, Doblhammer and Vaupel [2001] show the effect of the month of birth on mortality above age 50 caused by the nutrition of the

[16] This article has sparked a vigorous debate in the *International Journal of Epidemiology* between supporters of the conventional view of Brenner that improved socio-economic conditions improve health and supporters of Tapia Granados in line with Ruhm.

[17] Other economic indicators turn out to be insignificant.

[18] This results in a reduced body size or a thrifty metabolism adapted for poor nourishment, which increases the risk of cardiovascular and other diseases in adult life. The sex ratio is possibly also shifted toward more females. This programming constitutes a fast adaption mechanism within on generation supplementary to the evolutionary genetic adaption.

womb. With respect to the business cycle, significant negative effects of a good economic state at birth on mortality later on in life are found by van den Berg et al. [2006] for the birth cohorts 1812–1912 in the Netherlands.[19] Nevertheless, in this chapter we focus on short-run interactions of economic conditions and mortality and not on possible long-run consequences.

5.3 A Bayesian State Space Model

To capture the interrelation between mortality and macroeconomic time series, we use the methods described in Chapter 4. The common components of age-specific demographic variables are modeled as latent variables and linked with macroeconomic data through a structural vector autoregression (SVAR). To ensure smoothness along the age dimension, we assume for the coefficients which link the explanatory variables with the age-specific demographic variables, to follow auto regressive (AR) processes. In the following we describe the model in more detail.

The observed demographic variables $d_{x,t}$ with age classes $x = 0, \ldots, A$ and time periods $t = 1, \ldots, T$, can be expressed as

$$d_{x,t} \;=\; \overline{d}_x + \beta_x z_t + \epsilon^d_{x,t}\,, \tag{5.1}$$

with the arithmetic mean $\overline{d}_x = \frac{1}{T}\sum_{t=1}^{T} d_{x,t}$ and explanatory variables $z_t \equiv [\kappa_t \quad Y_t]'$, where κ_t is a $K \times 1$ vector of unobservables and Y_t is an $N \times 1$ vector of observed covariates. The corresponding coefficient vector $\beta_x \equiv [\beta^\kappa_x \quad \beta^Y_x]$ is $1 \times M$, where β^κ_x is a $1 \times K$ vector and β^Y_x is a $1 \times N$ vector with $M = K + N$. For the disturbances in Equation (5.1) we assume $\epsilon^d_{x,t} \sim i.i.d.\ \mathcal{N}(0, \sigma^2_d)$.

For the explanatory variables z_t we assume the vector autoregressive process

$$z_t = c + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \cdots + \phi_p z_{t-p} + \epsilon^z_t\,, \tag{5.2}$$

where c is an $M \times 1$ vector of constants and $\phi_1, \ldots, \phi_p$ are $M \times M$ coefficient matrices. The disturbances in Equation (5.2) can also be written as $\epsilon^z_t \equiv A\nu_t$, where A is an $M \times M$ coefficient matrix containing contemporaneous relations between the variables in z_t. For the $M \times 1$ vector of structural disturbances ν_t we assume $\nu^z_t \sim i.i.d.\ \mathcal{N}(0, I_M)$ and for the $M \times 1$ vector of reduced form disturbances ϵ^z_t we assume $\epsilon^z_t \sim i.i.d.\ \mathcal{N}(0, \Sigma_z)$,

[19] These findings are confirmed by van den Berg et al. [2008] for the Danish birth cohorts 1873–1906.

where $\Sigma_z = AA'$.

For the coefficient vector β_x we assume the law of motion

$$\beta_x = \alpha_1 \beta_{x-1} + \alpha_2 \beta_{x-2} + \cdots + \alpha_q \beta_{x-q} + \epsilon_x^\beta \,, \tag{5.3}$$

with $\epsilon_x^\beta \sim i.i.d.\ \mathcal{N}(0, \Sigma_\beta)$, where Σ_β is an $M \times M$ diagonal matrix. As the coefficient matrices $\alpha_1, \ldots, \alpha_q$ are also assumed to be diagonal, each component of β_x in fact follows an AR process on its own. All disturbances are assumed to be independent of each other.

To identify the model uniquely, we set the lower $K \times K$ block of β_x^κ to a diagonal matrix and the lower $K \times N$ block of β_x^Y to 0.

We choose the same priors as in Chapter 4. For the parameters in Equations (5.2)–(5.3) we assume Minnesota-type priors by centering the probability mass for the first lagged coefficient around 1 and for all subsequent lags around 0, whereby decreasing subsequently the uncertainty that the coefficients are 0 for more distant lags. For the variance of the disturbance in Equation (5.1) we assume a quite diffuse inverted gamma distribution.

5.4 Estimation

We estimate the model described in Equations (5.1)–(5.3) using MCMC methods. More precisely, we apply the Gibbs sampler. We draw from the joint distribution $\mathcal{P}(\Psi, z, \beta)$ by subdividing it into the conditional distributions $\mathcal{P}(\Psi \mid z, \beta)$, $\mathcal{P}(z \mid \Psi, \beta)$, and $\mathcal{P}(\beta \mid \Psi, z)$ and draw iteratively from them, where Ψ comprises all parameters of the model. Taken initialized values for $z^{(0)}$ and $\beta^{(0)}$ as given, we sample in the i-th iteration $\Psi^{(i)}$ from $\mathcal{P}(\Psi \mid z^{(i-1)}, \beta^{(i-1)})$, $z^{(i)}$ from $\mathcal{P}(z \mid \Psi^{(i)}, \beta^{(i-1)})$, and $\beta^{(i)}$ from $\mathcal{P}(\beta \mid \Psi^{(i)}, z^{(i)})$ successively. Under weak conditions and for $i \to \infty$ the Gibbs sampler converges and we obtain samples from the desired joint distribution $\mathcal{P}(\Psi, z, \beta)$.[20] For a more detailed description of the estimation procedure we refer to the technical Appendix of Chapter 4.

[20] See Geman and Geman [1984].

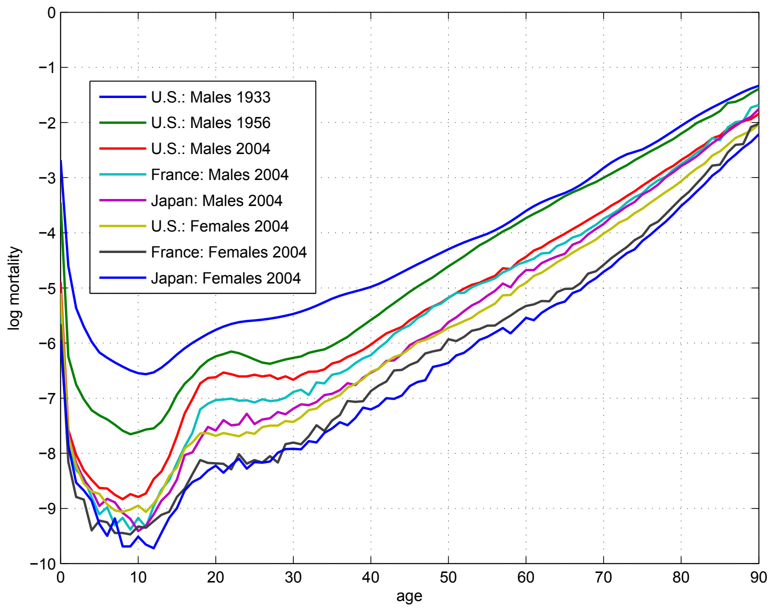

Figure 5.1: Age-specific log mortality for both sexes in the United States, France, and Japan.

5.5 Data

We analyze demographic-economic interactions in the United States in the periods 1956–2004 and 1933–1969 as well as in France and Japan in the period 1956–2004. The time series of logarithmized age-specific male and female mortality for 91 individual age classes from 0 to 90 are provided by the Human Mortality Database.[21] Figure 5.1 displays some examples of these age-specific mortalities.

As economic indicators for the business cycle we use time series of the unemployment rate and of GDP growth. The left column of Figure 5.2 displays the unemployment rates, which are measured as a percentage of unemployed in the civilian labor force aged 16 or older in the United States,[22] as standardized unemployment rate in France,[23] and as a percentage of unemployed in the labor force aged 15 or older in Japan.[24] The right column of Figure 5.2 displays the real GDP growth rates calculated from chained series

[21] See Human Mortality Database [2008]. In the Human Mortality Database obvious mistakes in the raw data are eliminated and death rates for the age classes 80 and above are smoothed by fitting a logistic function according to Thatcher et al. [1998] if the number of observations becomes too small. Wilmoth et al. [2007] supply a detailed method protocol. In the case of the United States, population estimates for 1940–1969 are adjusted and the extinct cohort method supposed by Kannisto [1994] is applied for the age classes 75 and above in the period 1933–1939. In the case of France, the data of infant deaths up to 1974 are corrected for false stillbirths.

[22] C.f U.S. Census Bureau [2007]. The pre 1947 unemployment figures refer to persons aged 14 or older, but this minor change causes no jump in 1947, when both definitions yield the same number.

[23] See OECD [2008].

[24] See Japan Statistics Bureau [2008b] and Japan Statistics Bureau [2008a].

provided by the Penn World Tables[25] and by the U.S. Census Bureau [2007] in case of the United States 1933–1969.

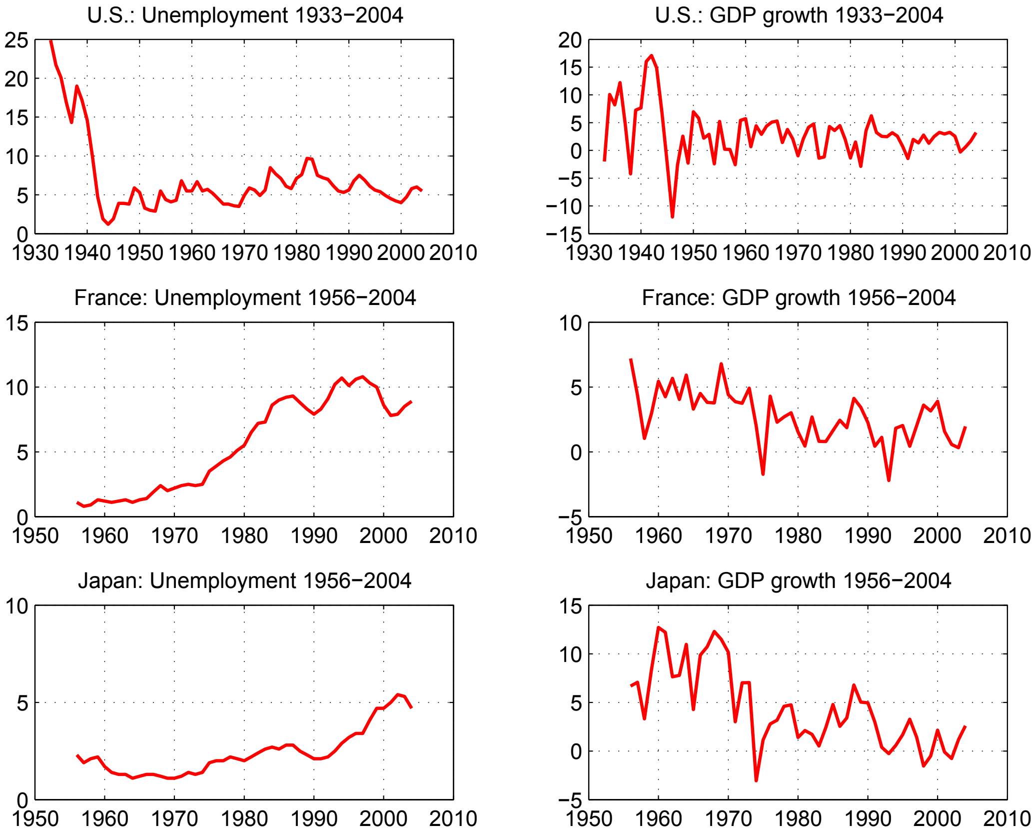

Figure 5.2: Unemployment rate and real GDP growth rate in the United States, France, and Japan. Note that the scales are different and use the uniform grid lines for convenient comparisons.

5.6 Empirical Results

For the empirical results we use a lag length of $p = 4$ for the z's and $q = 4$ for the β's. To ensure convergence of the Gibbs sampler we restart the algorithm several times using different starting values drawn from an overdispersed distribution and compare the results. We observe that our sampler reaches convergence already after a few thousand draws. To avoid influences of the starting values we discard the first half of the chain as burn-in phase.

In the following we begin our empirical analysis with data from the United States

[25] See Heston, Summers, and Aten [2006].

in the period 1956–2004 as our main application. Afterward, we compare the results with data for 1933–1969, in order to detect whether the relationship between macroeconomic variables and mortality rates evolves over time. Finally, we draw an international comparison with data from France and Japan.

5.6.1 Identification

We apply the Choleski-decomposition for the identification of structural shocks in the mutual impulse response functions of the three variables. This identification scheme builds on a triangularization of the covariance matrix Σ_z, which implies that not all variables can react instantaneously to an impulse in a particular variable. So, the ordering of the variables is crucial for the results. Of course, this also affects the presented responses of age-specific mortality which are derived from these mutual interactions. As already discussed in the introduction, mortality often reacts very quickly to short-run influences like changing economic conditions. In today's Western industrialized countries with their low and quite stable mortality, we assume the reverse effect to be small and less fast.[26] Thus, in the context of mortality and the business cycle, mortality has to be placed last to allow for instantaneous reactions to the economic variables placed before it. The main business cycle indicators GDP growth and unemployment are negatively correlated and the mutual effects are partly contemporaneous. Nevertheless, the unemployment rate is known as lagged business cycle indicator. The labor market is subject to many frictions and search and matching problems, which impede fast adjustments. Figure 5.2 shows that the unemployment rate is by far less volatile than the GDP growth rate. Hence, in the ordering of the variables

$$z_t \equiv [\, unemployment,\, GDPgrowth,\, \kappa\,]_t{}'$$

unemployment has to be placed before GDP growth to account for the more important instantaneous reactions of GDP growth on unemployment.

5.6.2 United States 1956–2004

First, we describe our empirical results for the post World War II United States data set. Figure 5.3 shows the surface of median mortality responses to an unemployment shock. The big differences regarding the responses of different age classes are an interesting aspect of this figure. While the 20 to 30 years olds react positively, all other age classes

[26]We do not consider possible major mortality crises caused by pandemics, large-scale natural disasters, war, terrorism, etc., but the small fluctuations related to the business cycle.

react negatively. As it is discussed below, these differences are found for responses to a GDP growth shock as well, suggesting that relevant information might get lost by crude age grouping instead of using single age class mortality rates.

There are two ways to illustrate our findings for a particular age class. First, we cut a plane parallel to the time axis out of the mortality surface described in Figure 5.3 and plot it with the corresponding error bands. We denote this as *age impulse response function* (AIRF). Second, we cut a plane out of the mortality surface along the diagonal of the age and time axis. We denote this as *cohort impulse response function* (CIRF). It turns out that the CIRFs and the AIRFs are very similar. Hence, for convenience we report the CIRFs only and state it explicitly when they deviate from each other. We construct CIRFs and AIRFs of age-specific female mortality rates as well. Since they are quite similar to the results we obtain for male mortality, we do not show the results for female mortality and state it explicitly when they are different from those for male mortality.

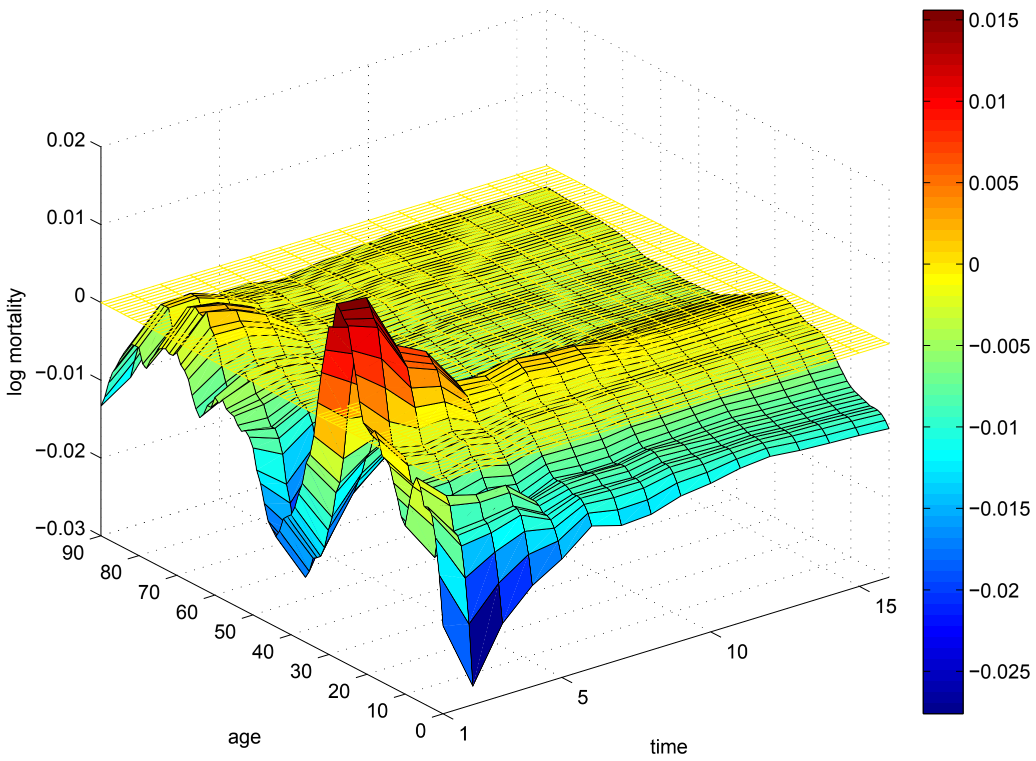

Figure 5.3: Surface of the median impulse response of logarithmized age-specific male mortality to a one standard deviation shock in unemployment in the United States 1956–2004. The time axis refers to the time elapsed since the impulse and the yellow grid marks the zero plane.

Figure 5.4 plots the male CIRF of an unemployment shock. Looking at the responses of child and adolescent mortality we find that the probability mass of the responses center around zero. For the age class of the 25 years olds we observe a positive response to an unemployment shock, which persists with most of the probability mass above zero up to the age of 28. For the age classes of 35 and 45 years olds we observe negative responses, which peter out after four years. For the age classes of 55 to 75 we find responses centered very much around zero. Finally, the age class of 85 years olds exhibits a negative contemporaneous reaction to an unemployment shock. Evidently, Figure 5.4 reveals that the age classes from 25 years olds to 45 years olds are strongly exposed to a shock in unemployment. This seems to be plausible, since a large part of the United States work force is included in this range. It should be noted that the anomaly we observe for the male mortality rates of the 25 years olds does not show up for female mortality rates.

Figure 5.5 reports the CIRF of a shock to GDP growth. The age group of 25 years

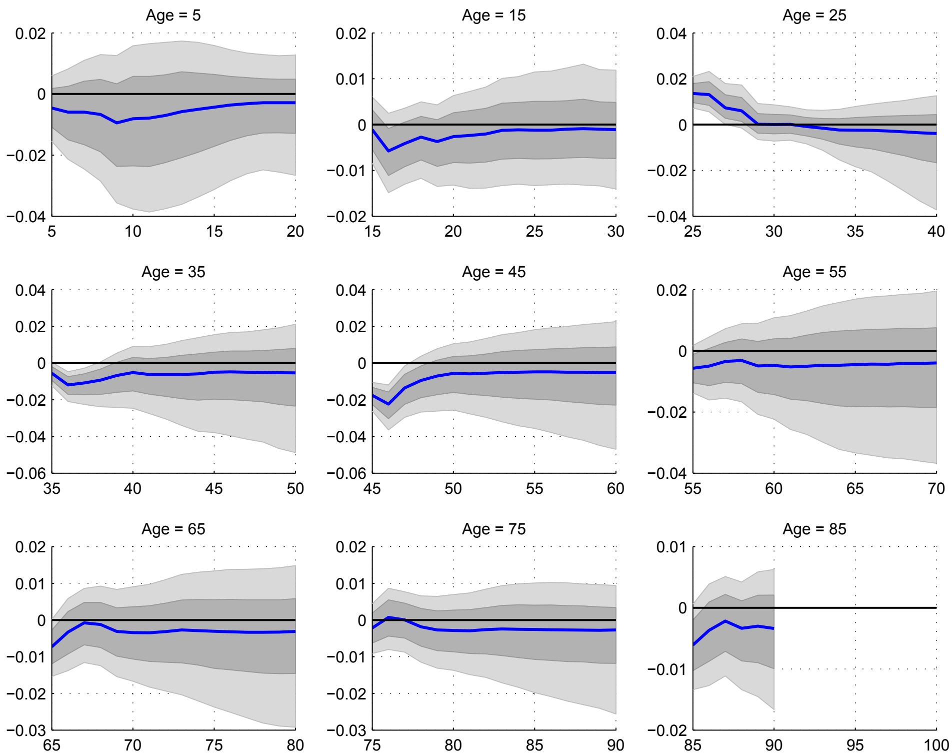

Figure 5.4: Impulse responses of log mortality in the further life of some male cohorts to a one standard deviation shock in unemployment occurring at the labeled age. The entire gray shaded area around the blue median represents 90% of the posterior probability mass and the dark gray shaded area represents 68% of the posterior probability mass.

olds represents an exception like in Figure 5.4. While all other age classes show a positive and persistent reaction, in the group of 25 years olds it is anticyclical in the beginning and turns positive after about five years. This indicates that the mortality responses of the 20 to 30 years olds are driven by different factors than that of all other age groups.

Overall, the findings presented in Figure 5.4 and 5.5 confirm for almost all age classes the evidence described in Ruhm [2000], whereas the reaction of male 25 years olds poses an exception, which is in line with the evidence found in e.g. Brenner [1971, 1979]. This discrepancy in the results for particular age classes implies that studies, which rely on crude age grouping, may miss important features of the data, because opposed effects possibly cancel each other. The anomaly in the mortality reactions of young adults coincides with the anomaly in the mortality level known as *accident hump*, which is associated with risky attitudes. According to this, a careless attitude of heavily discounting future consequences, leading to increased risk taking in the pessimism of an

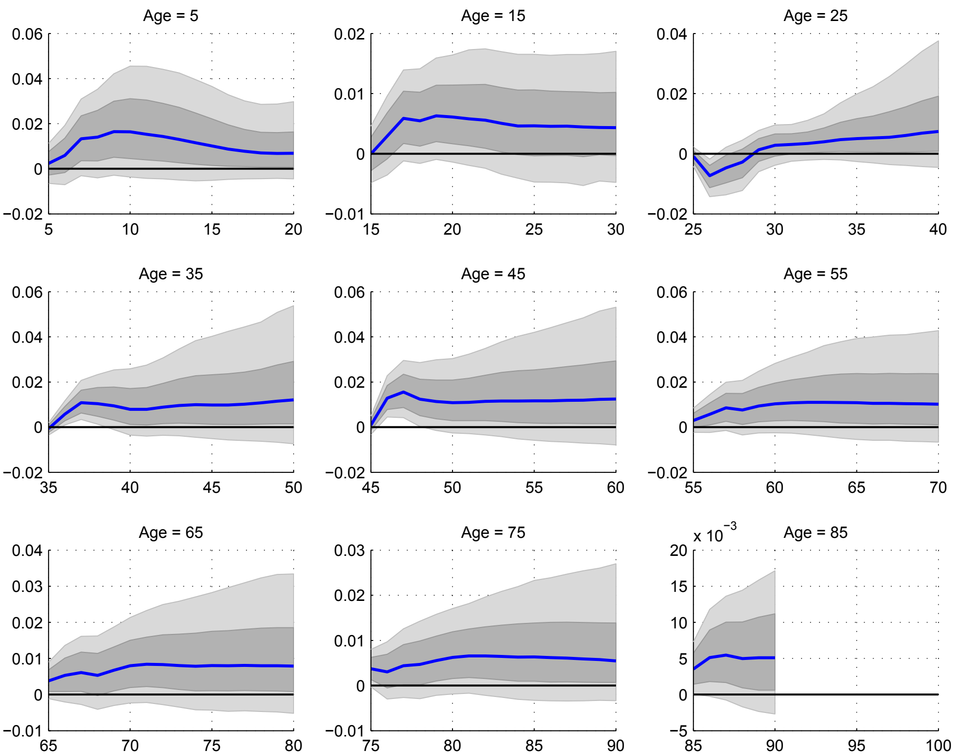

Figure 5.5: Impulse responses of log mortality in the further life of some male cohorts to a one standard deviation shock in GDP growth occurring at the labeled age. The entire gray shaded area around the blue median represents 90% of the posterior probability mass and the dark gray shaded area represents 68% of the posterior probability mass.

economic downturn, delivers a possible explanation for rising mortality of young adults. This is complemented by the possibility that they are in fact more exposed to economic hardship, because they have not yet accumulated sufficient resources to smooth consumption and may also suffer from stronger fluctuations of youth unemployment. The lack of an own family reinforces both arguments due to less responsibility and missing support. Another possible explanation for differential mortality is that chronic diseases are rare among young adults and, unlike older people, they do not suffer from increased numbers of infarctions triggered by stress in a boom. Hence, compared to the rest of the population, they may have health disadvantages in a recession, but advantages in a boom.

5.6.3 Change over Time: United States 1933–1969

The economy of the United States underwent dramatical changes during the period 1933–2004. Some major events, possibly altering the relationship between macroeconomic variables and mortality, are the Great Depression of the early 1930s and World War II. Looking at the upper-left panel of Figure 5.2 we observe an enormous decline of the unemployment rate in the Unites States during the 1930s and 1940s, indicating that the decreasing unemployment and mortality rates coincide. To analyze these event separately we make use of the sub-period 1933–1969 and construct CIRFs.

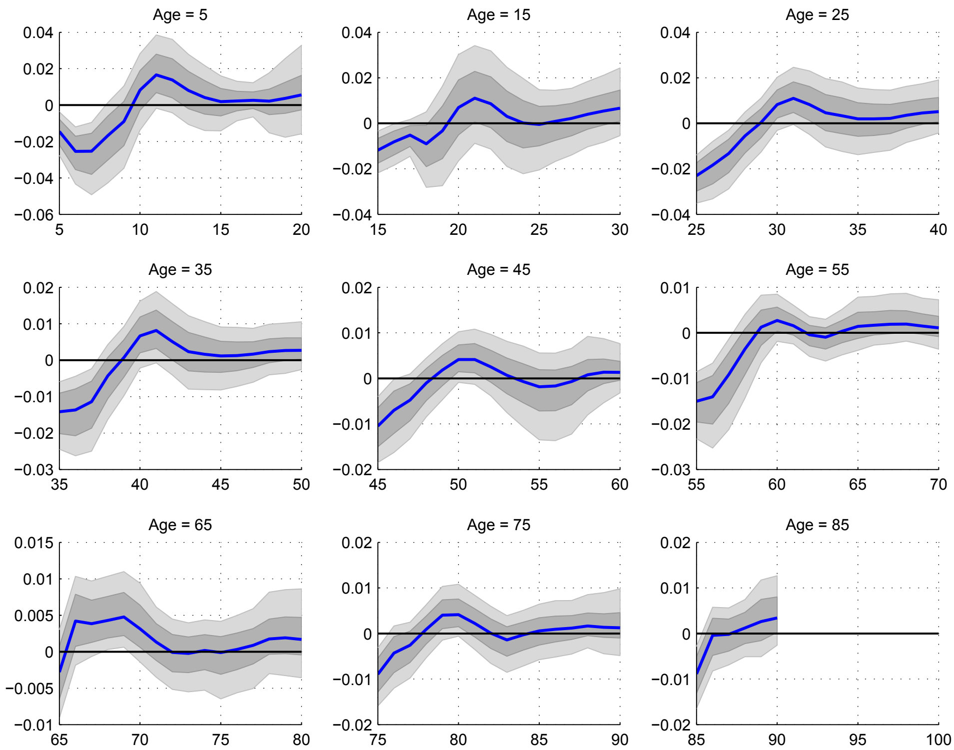

Figure 5.6: Impulse responses of log mortality in the further life of some male cohorts in the period 1933–1969 to a one standard deviation shock in unemployment occurring at the labeled age. The entire gray shaded area around the blue median represents 90% of the posterior probability mass and the dark gray shaded area represents 68% of the posterior probability mass.

Figure 5.6 and 5.7 report the CIRFs for the period 1933–1969. As opposed to the post World War II period, all age classes react in a similar fashion to a shock in unemployment and GDP growth. While all age classes react negatively to a shock in unemployment in the first four years, all responses switch signs and turn positive afterward. The responses to a shock in GDP growth are positive and last about five years. Two outcomes in Figure 5.6 and 5.7 are striking. First, on the one hand Figure 5.6 indicates that the short term responses of the mortality rates are similar to results found in Ruhm

[2000].[27] On the other hand it also indicates that the mid-term responses are line with e.g. Brenner [1971, 1979]. Second, whereas the response of the age class of the 25 years olds is procyclical in the 1954–2004 period, it was anticyclical for 1933–1969. This suggests that transmission channels from macroeconomic variables to age-specific mortality rates might have changed over time.

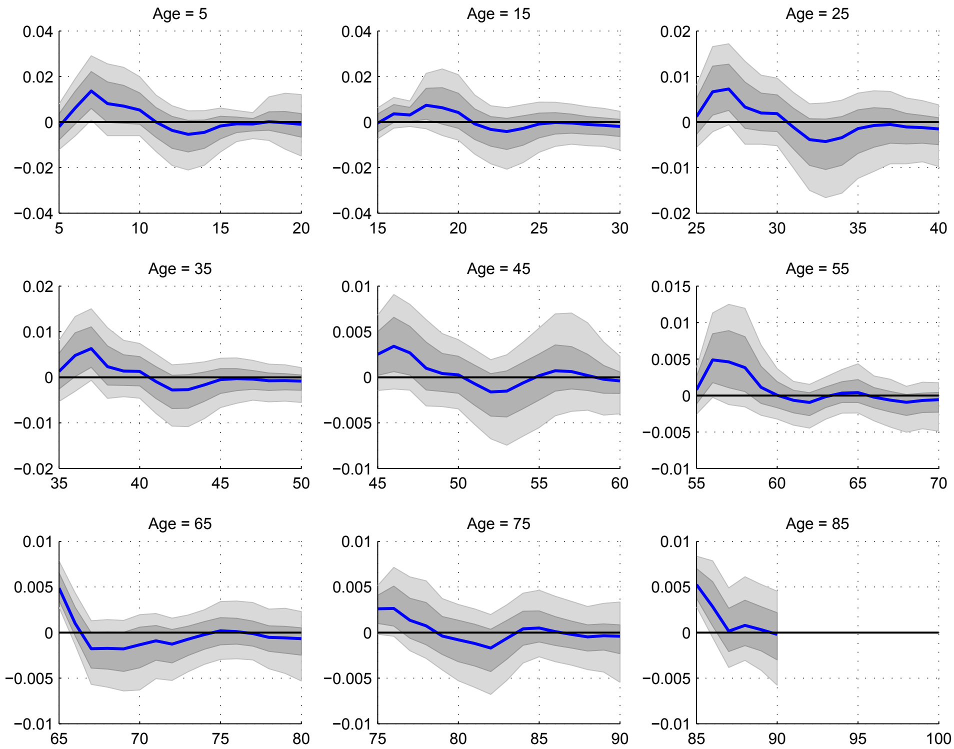

Figure 5.7: Impulse responses of log mortality in the further life of some male cohorts in the period 1933–1969 to a one standard deviation shock in GDP growth occurring at the labeled age. The entire gray shaded area around the blue median represents 90% of the posterior probability mass and the dark gray shaded area represents 68% of the posterior probability mass.

5.6.4 International Comparison: France and Japan 1956–2004

The influence of the business cycle on mortality may not only vary by time, but also across countries. To draw an international comparison, we analyze data from France and Japan in the period 1956–2004 in addition to the United States. These examples of Western industrialized countries from three continents differ in many economic, institutional, and demographic aspects, so that they are well suited to detect possible

[27]See Section 5.2.2

differences, but also common features in the interrelation of economic fluctuations and mortality.[28]

In fact, the reactions patterns after an unemployment shock for France and Japan are quite different from that for the United States. Figure 5.8 and 5.9 present male CIRFs for these countries. The main common feature in all three countries is the temporarily increased mortality of young adults around age 25. In France and Japan, the rise actually appears for both sexes and in a wider age range than in the United States. The reactions of most of the other age classes exhibit definite discrepancies. Unlike to the United States, no patterns of pronounced short-lived negative responses in the age groups 35 and 45 exist. Most of the probability mass for the 35 years olds is temporarily above zero, which is in line with the mortality responses of the even younger adults. The mortality reactions of all age classes between 45 and 85 are clearly and persistently negative in France. In Japan, at age 65 most of the probability mass is below zero, too, and the reactions are clearly negative at the highest ages of 75 and 85.

A special feature for male children in France illustrates possible differences between the mortality reactions at a fixed age (AIRF) and of an aging cohort (CIRF). The mortality reaction of 5 years olds in the AIRF is persistently negative.[29] After the first years, in which it is centered around zero, most of the probability mass of the response at age 15 is below zero, too. In contrast, the responses in the presented CIRF get increasingly positive, when the originally 5 or 15 years olds enter the phase of young adulthood. This means that people can be subject to particular long-lasting mortality reactions in an age group, even if they enter this age group not until several years after the shock. Whereas these people have at least experienced the shock in an earlier phase of their life cycle, the finding from the AIRF actually implies that male children in France still profit from an unemployment shock many years before their own birth. In this case, the transmission to the children probably proceeds via lasting changes in the characteristics of parents and social environment. Of course, child mortality in today's industrialized countries is very low, so that even very little absolute changes matter a lot.[30] It is noteworthy that mortality decreases after an unemployment shock are restricted to male children. At the age of 15, female mortality exhibits an increase in France and in Japan.

[28]Figure 5.1 shows that the level of current mortality at almost all age classes is highest in the United States and lowest in Japan.

[29]For the AIRF, see Figure 10 in the appendix.

[30]In each single age class of males between 5 and 12 only 30–50 cases of death occurred in France in 2004. The numbers for females are even lower. See Human Mortality Database [2008].

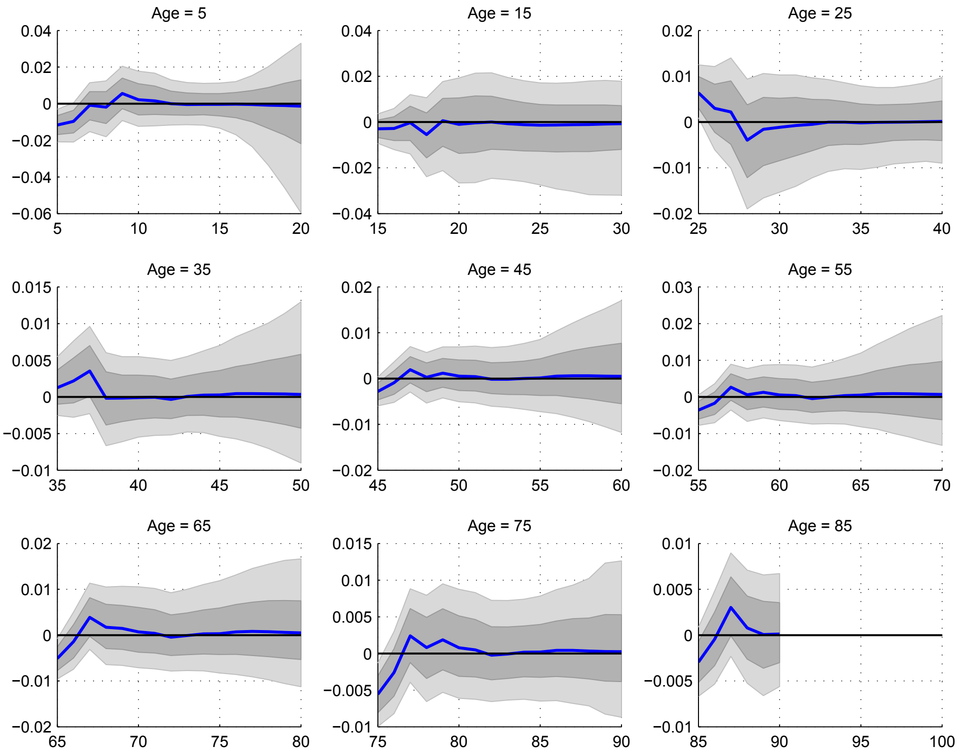

Figure 5.8: Impulse responses of log mortality in the further life of some male cohorts in France to a one standard deviation shock in unemployment occurring at the labeled age. The entire gray shaded area around the blue median represents 90% of the posterior probability mass and the dark gray shaded area represents 68% of the posterior probability mass.

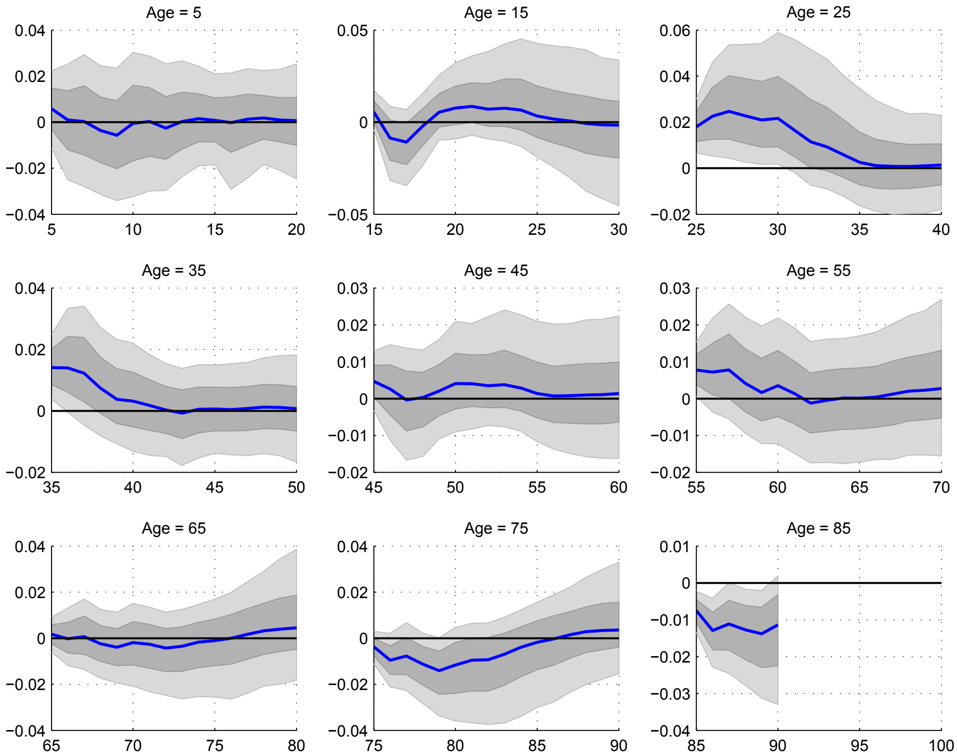

Figure 5.9: Impulse responses of log mortality in the further life of some male cohorts in Japan to a one standard deviation shock in unemployment occurring at the labeled age. The entire gray shaded area around the blue median represents 90% of the posterior probability mass and the dark gray shaded area represents 68% of the posterior probability mass.

The patterns of mortality reactions after a shock in GDP growth are relatively similar in all three countries.[31] Most age classes show a persistent increase, at least after the first one or two years. Only young adults in France and the United States exhibit decreased mortality. In the United States, the effect is short-lived and restricted to the male age class of 25 years olds. In France, the effect lasts a little bit longer; and for 15 years olds after several years and for females in the first year, most of the probability mass is below zero, too.

All in all, the international comparison yields some differences, but as most striking result that in most cases a distinction has to be made between young adults and the rest of the population. With the exception of females in the United States, young adults always suffer from increased unemployment. On the other hand, they often profit from increased GDP growth. Thus, their mortality is low in a boom and high in a recession.

[31]See Figures 11 and 12 in the appendix.

The rest of the population does not always react conclusively, but often profits from increased unemployment. These reactions are quite short in the United States and most widespread and persistent in France.[32] GDP growth always turns out to be harmful. Hence, for a large fraction of the population mortality is high in a boom and low in a recession.

5.6.5 Ethical Dilemma

The detected relationship with procyclical mortality of a large part of the population is a delicate issue. While young adults often profit, many people suffer at the same time from a small, but significant increase of mortality in a boom. This might be seen as ethical dilemma, because a good state of the economy is in general assessed as desirable. However, we think that this desirability still holds even if an economic upturn slightly increases mortality. From a theoretical point of view, which is most prevalent in economics, rational economic agents always make their decisions with the objective to maximize their own expected utility. The methodological individualism implicates that a high output and a high employment as well as all their direct and indirect implications result from these voluntary choices. Consequentially, the overall outcome has to be associated with high utility in average and is desired by the agents.[33] From an empirical point of view, there is also no contradiction in aiming for a boom despite its mortality side effect, because people uncoerced engage in all kinds of risky behavior in their private life, too. They ignore safety and health advices of all kinds and often trade off a slightly increased risk of death against various benefits in their own pursuit of happiness.[34] Nevertheless, with respect to both points of view, individuals might suffer from restricted information distorting their decisions. Hence, we think that in principle there is a role for health education to mitigate the adverse mortality effect of changed living conditions and individual behavior along the business cycle.

5.7 Conclusion

In this chapter we analyze the impact of short-run economic fluctuations on age-specific mortality. We contribute to the debate on the cyclicality of the effects of the business cycle on mortality triggered by recent findings of procyclical mortality by Ruhm [2000]

[32] This is of course in line with the more flexible labor market in the United States

[33] Admittedly, there might be problems with external effects of individually optimal choices.

[34] Many people like to consume alcohol or tobacco, have an unhealthy diet, do hazardous sports, participate unnecessarily in traffic, visit dangerous places, or expose themselves to sexually transmitted diseases.

which contradict conventional socio-epidemiological wisdom according to e.g. Brenner [1971, 1979].

For the first time, we examine the differing consequences of economic changes for all individual age classes. To this end, we build on the model of Chapter 4 to set up structural VARs of a latent mortality variable and of unemployment and GDP growth as main business cycle indicators. Age-specific coefficients link these variables to the actual mortality of single age classes. Impulse response analyses show the age-specific mortality reactions to structural shocks in the economic variables.

For the United States in the period 1956–2004, we find that young male adults noticeably differ from the rest of the population. The 25 years olds exhibit increased mortality in a recession, whereas the age classes of 35 or 45 years olds react with lower mortality to increased unemployment and, like older people, with higher mortality to increased GDP growth. Thus, analyses of the cyclicality of mortality changes have to differentiate closely between particular age classes, especially in the age range of young adults, to avoid spurious results due to possible neutralization of opposed effects. The special role of young people with respect to mortality changes coincides with that in mortality levels known as *accident hump*. Possible explanations for the anomaly in the mortality reactions span from higher risk taking or actually more severe adverse effects on young adults in a recession to chronic diseases of older people facilitating acute myocardial infarctions in the stress of a boom.

In an analysis of an earlier period since 1933, all age classes react procyclically to economic changes. This may point to long-term changes of the channels between macroeconomic conditions and mortality. Admittedly, the aftermath of the Great Depression dominates this sample. An international comparison with France and Japan confirms the special role of young adults in the post-war period. Their countercyclical pattern of mortality reactions extends to both sexes and affects even a little wider age range than in the United States. Most other age classes show a procyclical reaction pattern. This also holds for people in the retirement age, in particular outside of the United States, who are not directly subject to the state of the labor market. In general, the mortality responses are most short-lived in the United States and most persistent in France, which suggests a relation to institutional differences. Nevertheless, the clear-cut contrast between countercyclical mortality of young adults and widespread procyclical mortality in the rest of the population is the most striking result.

Appendix to Chapter 1

1 Estimation procedure

1.1 Estimating the Parameters

In this section we condition on the factor f_t and the factor loadings Λ_t. We follow Chib [1993] and Kim and Nelson [1999a].

We begin by rewriting equation (2.3) as:

$$u_i = X_{i,u}\theta_i + \chi_i \tag{4}$$

where $u_i = [u_{i,p+1}\ u_{i,p+2}\ \dots\ u_{i,T}]'$ is $T - p \times 1$, $\theta_i = [\theta_{i,1}\ \theta_{i,2}\ \dots\ \theta_{i,p}]'$, is $p \times 1$ and $\chi_i = [\chi_{i,p+1}\ \chi_{i,p+2}\ \dots\ \chi_{i,T}]'$ is $T - p \times 1$ and

$$X_{i,u} = \begin{bmatrix} u_{i,p} & u_{i,p-1} & \cdots & u_{i,1} \\ u_{i,p+1} & u_{i,p} & \cdots & u_{i,2} \\ \vdots & \vdots & \vdots & \vdots \\ u_{i,T-1} & u_{i,T-2} & \cdots & u_{i,T-p} \end{bmatrix}$$

which is a $T - p \times p$ for $i = 1, 2, \dots, N$.

Combining the priors described in section 2.2.2 with the likelihood function we obtain the following posterior distributions.

The posterior of the AR-parameters of the idiosyncratic components is:

$$\theta_i \sim N(\overline{\theta_i}, \overline{V}_{i,\theta})I_{S_\theta} \tag{5}$$

where

$$\overline{\theta_i} = \left(\underline{V}_\theta^{-1} + (\sigma_{i,\chi}^2)^{-1}X_{i,u}'X_{i,u}\right)^{-1}\left(\underline{V}_\theta^{-1}\underline{\theta} + (\sigma_{i,\chi}^2)^{-1}X_{i,u}'u_i\right)$$

and

$$\overline{V}_{i,\theta} = \left(\underline{V}_{\theta}^{-1} + (\sigma_{i,\chi}^2)^{-1} X_{i,u}' X_{i,u}\right)^{-1}.$$

where I_{S_θ} is an indicator function enforcing stationarity.

The posterior of the variance of the idiosyncratic component $\sigma_{i,\chi}$ is:

$$\sigma_{i,\chi}^2 \sim \mathcal{IG}\left(\frac{(T+\alpha_\chi)}{2}, \frac{((u_i - X_i\theta_i)'(u_i - X_i\theta_i) + \delta_\chi)}{2}\right) \tag{6}$$

The posterior of the variance of the factor loadings $\sigma_{i,\epsilon}$ is:

$$\sigma_{i,\epsilon}^2 \sim \mathcal{IG}\left(\frac{(T+\alpha_\epsilon)}{2}, \frac{((\Delta\lambda_i)'(\Delta\lambda_i) + \delta_\epsilon)}{2}\right) \tag{7}$$

where $\lambda_i = [\lambda_{i,1} \ \lambda_{i,2} \ \ldots \ \lambda_{i,T}]'$ and Δ is the first difference operator for this vector. To estimate the AR-parameters of the factor $\varphi_1, \varphi_2, \ldots, \varphi_q$ we find it useful to rewrite equation (2.2) as:

$$f = X_f\varphi + \nu \tag{8}$$

where $f = [f_{q+1} \ f_{q+2} \ \ldots \ f_T]'$ is $T - q \times 1$, $\varphi = [\varphi_1 \ \varphi_2 \ \ldots \ \varphi_q]'$ is $q \times 1$, $\nu = [\nu_{q+1} \ \nu_{q+2} \ \ldots \ \nu_T]'$ is $T - q \times 1$ and

$$X_f = \begin{bmatrix} f_q & f_{q-1} & \cdots & f_1 \\ f_{q+1} & f_q & \cdots & f_2 \\ \vdots & \vdots & \vdots & \vdots \\ f_{T-1} & f_{T-2} & \cdots & f_{T-q} \end{bmatrix}$$

which is $T - q \times q$. Thus, the posterior of the AR-parameters of the factor is:

$$\varphi \sim N(\overline{\varphi}, \overline{V}_\varphi) I_{S_\varphi} \tag{9}$$

where

$$\overline{\varphi} = \left(\underline{V}_{\varphi}^{-1} + (X_f' X_f)\right)^{-1} \left(\underline{V}_{\varphi}^{-1} \underline{\varphi} + (X_f' f)\right)$$

and

$$\overline{V}_f = \left(\underline{V}_{\varphi}^{-1} + X_f' X_f\right)^{-1}.$$

where I_{S_φ} is an indicator function enforcing stationarity.

To estimate the factor loadings, when they are assumed to be constant, we rewrite equation (2.1) as:

$$y_i^* = \lambda_i f^* + \chi \tag{10}$$

where $y_i^* = [(1-\theta(L)_i)y_{i,p+1}\ (1-\theta(L)_i)y_{i,p+2} \dots\ (1-\theta(L)_i)y_{i,T}]'$ which is $T-p\times 1$ and $f^* = [(1-\theta(L)_i)f_{p+1}\ (1-\theta(L)_i)f_{p+2}\ \dots\ (1-\theta(L)_i)f_T]'$, which is $T-p\times 1$ with $\theta(L)_i = (\theta_{i,1}+\theta_{i,2}+\cdots+\theta_{i,p})$ for $i=1,2,\dots,N$. Thus, the posterior for the constant factor loadings is:

$$\lambda_i \sim N(\overline{\lambda_i}, \overline{V}_{i,\lambda}) \tag{11}$$

where

$$\overline{\lambda_i} = \left(\underline{V}_\lambda^{-1} + (\sigma_{i,\chi}^2)^{-1} f^{*\prime} f^*\right)^{-1} \left(\underline{V}_\lambda^{-1}\underline{\lambda} + (\sigma_{i,\chi}^2)^{-1} f^{*\prime} y_i^*\right)$$

and

$$\overline{V}_{i,\lambda} = \left(\underline{V}_\lambda^{-1} + (\sigma_{i,\chi}^2)^{-1} f^{*\prime} f^*\right)^{-1}.$$

1.2 Estimating the Latent Factor

To estimate the common latent factor we condition on the parameters of the model $\Xi \equiv (\varphi_1, \varphi_2, \dots, \varphi_q, \Theta_1, \Theta_2, \dots, \Theta_p)$ and the factor loadings Λ_t. We follow Kim and Nelson [1999a].

We begin by quasi-differencing equation (2.1) and use it as our observation equation in the following state-space system:

$$Y_t^* = H_t F_t + \chi_t \tag{12}$$

where

$$Y_t^* = (\mathcal{I}_N - \Theta(L))Y_t$$

$$H_t = [\Lambda_t\ \ -\Theta_1\Lambda_{t-1}\ \ -\Theta_2\Lambda_{t-2}\ \dots\ \Theta_p\Lambda_{t-p}\ 0_{N\times q-p-1}]$$

with

$$\Theta(L) = (\Theta_1 + \Theta_2 + \cdots + \Theta_p)$$

Our state equation is:

$$F_t = \Phi F_{t-1} + \widetilde{\nu}_t \tag{13}$$

where $F_t = [f_t, f_{t-1}, \ldots, f_{t-q+1}]'$ is $q \times 1$, which is denoted as the state vector, $\widetilde{\nu}_t = [\nu_t \; 0 \; \ldots \; 0]'$ is $q \times 1$ and

$$\Phi = \begin{bmatrix} \varphi_1 & \varphi_2 & \cdots & \varphi_q \\ & \mathcal{I}_{q-1} & & 0_{q-1\times 1} \end{bmatrix}$$

which is $q \times q$. For all empirical results shown below we use $q > p$.

To calculate the common factor we use the algorithm suggested by Carter and Kohn [1994] and Frühwirth-Schnatter [1994]. The vector $F = [F_1 \; F_2 \; \ldots \; F_T]$ can now be drawn from its joint distribution given by:

$$p(F|\Lambda, Y, \Xi) = p(F_T|\Lambda_T, y_T, \Xi) \prod_{t=1}^{T-1} p(F_t|F_{t+1}, \Lambda_t, \Xi, Y^t) \tag{14}$$

where $\Lambda = [\Lambda_1 \; \Lambda_2 \; \ldots \; \Lambda_T]$ and $Y^t = [Y_1 \; Y_2 \; \ldots \; Y_t]$. Because the error terms in equations (12) and (13) are Gaussian equation (14)can be rewritten as:

$$p(F|\Lambda, Y, \Xi) = N(F_{T|T}, P_{T|T}) \prod_{t=1}^{T-1} N(F_{t|t,F_{t+1}}, P_{t|t,F_{t+1}}) \tag{15}$$

with

$$F_{T|T} = E(F_T|\Lambda, \Xi, Y) \tag{16}$$

$$P_{T|T} = Cov(F_T|\Lambda, \Xi, Y) \tag{17}$$

and

$$F_{t|t,F_{t+1}} = E(F_t|F_{t+1}, \Lambda, \Xi, Y) \tag{18}$$

$$P_{t|t,F_{t+1}} = Cov(F_t|F_{t+1}, \Lambda, \Xi, Y) \tag{19}$$

We obtain $F_{T|T}$ and $P_{T|T}$ from the last step of the Kalman filter iteration and use them as the conditional mean and covariance matrix for the multivariate normal distribution $N(F_{T|T}, P_{T|T})$ to draw F_T. To illustrate the Kalman Filter we work with the state-space system equations (12) and (13). We begin with the prediction steps:

$$F_{t|t-1} = \Phi F_{t-1|t-1} \tag{20}$$

$$P_{t|t-1} = \Phi P_{t-1|t-1}\Phi + Q \tag{21}$$

where

$$Q = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

which is $q \times q$. To update these predictions we first have to derive the forecast error:

$$\kappa_t = Y_t^* - H_t F_{t|t-1} \tag{22}$$

its variance:

$$\Sigma = H_t P_{t|t-1} H_t' + \Omega_\chi \tag{23}$$

and the Kalman gain:

$$K_t = P_{t|t-1} H_t' \Sigma^{-1}. \tag{24}$$

Thus, the updating equations are:

$$F_{t|t} = F_{t|t-1} + K_t \kappa_t, \tag{25}$$

$$P_{t|t} = P_{t|t-1} + K_t H_t P_{t|t-1}, \tag{26}$$

To obtain draws for $F_1, F_2, \ldots, F_{T-1}$ we sample from $N(F_{t|t,F_{t+1}}, P_{t|t,F_{t+1}})$, using a backwards moving updating scheme, incorporating at time t information about F_t contained in period $t+1$. More precisely, we move backwards and generate F_t for $t = T-1, \ldots, p+1$ at each step while using information from the Kalman filter and F_{t+1} from the previous step. We do this until $p+1$ and calculate $f_1, f_2, \ldots, f_p$ in an one-step procedure.

The updating equations are:

$$F_{t|t,F_{t+1}} = F_{t|t} + P_{t|t}\Phi' P_{t+1|t}^{-1}(F_{t+1} - F_{t+1|t}) \tag{27}$$

and

$$P_{t|t,F_{t+1}} = P_{t|t} - P_{t|t}\Phi' P_{t+1|t}^{-1}\Phi P_{t|t} \tag{28}$$

1.3 Estimating the Time-Varying Factor Loadings

To estimate the time-varying factor loadings we condition on the parameters Ξ and the factor f_t. We follow Del Negro and Otrok [2003]. Because equation (2.1) and equation (2.4) are N independent linear regressions, the factor loadings can be estimated equation by equation. Hence, we use the following state-space system and begin with the observation equation

$$y^*_{i,t} = z_{i,t}\widetilde{\lambda}_{i,t} + \chi_{i,t} \tag{29}$$

where $y^*_{i,t} = (1 - \theta(L)_i)y_{i,t}$, $z_{i,t} = [f_t \ \ -\theta_{i,1}f_{t-1} \ \ldots \theta_{i,p}f_{t-p}]$, which is $1 \times p + 1$, $\widetilde{\lambda}_{i,t} = [\lambda_{i,t} \ \lambda_{i,t-1} \ \ldots \ \lambda_{i,t-p}]'$, which is $p + 1 \times 1$ and with $\theta(L)_i = (\theta_{i,1} + \theta_{i,2} + \cdots + \theta_{i,p})$ for $i = 1, 2, \ldots, N$.

The state equation is

$$\widetilde{\lambda}_{i,t} = A\widetilde{\lambda}_{i,t-1} \tag{30}$$

where

$$A = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ & \mathcal{I}_p & & 0_{p\times 1} \end{bmatrix}$$

which is $p + 1 \times p + 1$. After we have defined the state-space system, calculating the time-varying factor loadings is straightforward as we just have to apply the Carter and Kohn [1994] and Frühwirth-Schnatter [1994] algorithm described above.

Because $\widetilde{\lambda}_{i,t}$ follows a driftless random walk and hence is not a stationary process it is not possible to use the unconditional mean and variance as starting values for the Kalman filter anymore [Hamilton, 1994, 378]. Thus, we decided to use the estimates for the constant factor loadings as a proxy for the initial conditions[35].

[35]We applied this to simulated data and obtained very satisfying results.

2 Series and Sources

Table 1: Data and Sources

	Series	Code	Units	98	53
1	Cargo moved on NY State canals	Df696	short tons	x	x
2	U.S. Tea Imports	m07040	mio pounds	x	
3	Prod. of Nonfarm Resid. Housekeeping Units	a02238	nr of units produced	x	
4	Nonfarm Nonresid. Building Activity	a02240	mio current dollars	x	
5	Total Nonfarm Building Activity	a02241	mio current dollars	x	
6	Live Hog Receipts	m01038	thousands of head	x	
7	Rail Consumption	a02084	1000 long tons	x	
8	Merchant Vessels	a02244	gross tons	x	
9	Building Permits, Chicago	a02047	mio current dollars	x	
10	Merchant Marine	a02135	1000 gross tons	x	
11	Yachts Built	a02102	gross tons	x	
12	Nonfarm Resid. Building Activity	a02239	mio current dollars	x	
13	Raw Silk Imports	m7037a-c	thousands of tons	x	
14	Coffee Imports	m07038	mio of pounds	x	
15	Tin Imports	m07042	long tons	x	
16	Raw Cotton Exports	m07043a	mio of pounds	x	
17	Miles of Railroad Built	a02082a	miles	x	
18	Nr. of Concerns in Business	a10030	thousands	x	
19	Index of US Business Activity	m12003	percent of trend	x	
20	Bank Clearings	m12015	Daily Average	x	
21	Wholesale Price Cotton, raw	m04006a	cents per pound	x	x
22	Whs. Price of Wheat, Chicago, 6 Markets	m04001a	cents per bushel	x	
23	Wholesale Price of Corn Chicago	m04005	dollars per bushels	x	
24	Wholesale Price of Cattle Chicago	m04007	dollars per hundred pounds	x	
25	Wholesale Price of Hogs Chicago	m04008	1000 tons	x	
26	Copper Prices	Cc253-258	Dollars per pound	x	
27	Brick Prices	Cc264-266	dollars per thousand	x	
28	Prices of Anthr. Foundry Pig Iron	m04011a	dollars per ton of 2240 lbs.	x	
29	Whs. Price of Copper	m04015a	cents per pound	x	
30	Total Exports	m07023	mio of dollars	x	
31	Total Imports	m07028	mio of dollars	x	
32	Earnings Yield NYSE Common Stocks	a13049	%	x	
33	Index of Whs. Prices	Cc125		x	
34	Index General Price Level	m04051	cents per pound	x	
35	Call Money Rates Mixed Coll.	m13001	%	x	
36	Am. Railroad Bond Yields	m13019	%	x	
37	National Bank Notes Outst.	m14124a	mio of dollars	x	
38	Comm. Paper Rates NY City	m13002	%	x	
39	Oats production	Da667-678	Thousand metric tons	x	x
40	Cotton production	Da755-765	Thousand short tons	x	x
41	Raw steel production	Dd399	Thousand short tons	x	
42	Patents granted	Cg38	Number	x	x
43	Stock Prices	Cj797*	1802=10	x	x
44	US Notes	Cj60	thousand dollars	x	x

Overview cont'd

	Series	Code	Units	98	53
45	Business Failures	Ch411	Number	x	x
46	Coal Fuel Mineral Production	Db25-33	Thousand short tons	x	x
47	Vessels entered US ports	Df594	thousand net tons	x	x
48	Wool Prices	Cc226-230	Dollars per pound	x	x
49	Coal Prices	Cc235-242	Dollars per ton of 2240 lbs.	x	x
50	Irish potatoes Acreage	Da 768	Thousand acres	x	x
51	Irish potatoes Production	Da 769	Thousand tons	x	x
52	Irish potatoes price	Da 770	dollars per hundred weight	x	x
53	Cattle Nr	Da 968	Number	x	x
54	Cattle Price	Da 969	Value per head	x	x
55	Hogs Nr	Da 970	Number	x	x
56	Hogs Price	Da 971	Value per head	x	x
57	Cows and heifers	Da1020	Number	x	x
58	Cows and heifers	Da 1021	Value per head	x	x
59	Butter Price	Da 1036	Cents per pound	x	
60	Petroleum Price	Db 56	Average value at well	x	x
61	Bit. Coal Production	Db 60	Thousand short tons	x	x
62	Bit. Coal Imports for Consumption	Db 64	Thousand short tons	x	
63	Bit Coal Exports	Db 65	Thousand short tons	x	
64	Pig iron shipments	Db 74	Thousand short tons	x	x
65	Production from mines	Db 75	metric tons	x	x
66	Lead production	Db 80	metric tons	x	x
67	Zinc production	Db 84	metric tons	x	x
68	Gold production	Db 94	kg	x	x
69	Silver production	Db 95	metric tons	x	x
70	Refined lead imports	Db 146	metric tons	x	x
71	Coal Exports	Db 191	Thousand short tons	x	x
72	Wheat flour	Dd 368	Thousand short tons	x	x
73	Hot rolled iron and steel	Dd 405	Thousand short tons	x	x
74	Rails	Dd 407	Thousand short tons	x	x
75	Corn/Harvested for grain	Da 697	Acreage Harvested	x	x
76	Coffee, imported	Dd843	Million pounds	x	x
77	Telegraph Operating Revenues	Dg 19 / 18	Million dollars	x	
78	Barley acreage harvested	Da701	Thousand acres	x	x
79	Barley Production	Da702	Thousand bushels	x	x
80	Flaxseed	Da705	Dollars per hundredweight	x	x
81	Exports of merchandise, gold, and silver	Ee362	Dollars	x	x
82	Imports of merchandise, gold, and silver	Ee363	Dollars	x	x
83	Exports and Imports	Ee1	Million dollars	x	x
84	Merchandise Imports and Duties	Ee 425	Dollars	x	x
85	Cotton, unman. exports	Ee571	Million dollars	x	
86	Tea Imports	Ee594	Cents per pound	x	
87	Sugar Imports	Ee596	Dollars per barrel	x	
88	All wheat acreage	Da717	thousand acres	x	x
89	All wheat production	Da718	million bushels	x	x
90	All wheat price	Da719	dollars per bushels	x	x
91	Hay acreage	Da733	Thousand acres	x	x
92	Hay production	Da734	Thousand bushels	x	x
93	Hay price	Da735	Dollars per short ton	x	x

Overview cont'd

	Series	Code	Units	98	53
94	Rye acreage	Da740	Thousand acres	x	x
95	Rye production	Da741	Thousand bushels	x	x
96	Rye price	Da742	dollars per bushel	x	x
97	Net Savings of Life Ins. Policy Holders	a10036a	Million dollars	x	
98	Population	Aa7	Thousand	x	x

*from 1871-1896: Cowles Comm. (m11025a). 1867-1870: Railroad stocks (m11005).

Source: A-, C-, D-, E-codes: Historical Statistics of the United States (Carter et al., 2006)
a-, m-codes: NBER macro history database

Appendix to Chapter 2

3 Estimation procedure

3.1 Estimating the Parameters

This section conditions on the factors U_t to calculate the parameters of the model.[36]

Rewriting equation (3.3) as

$$v_j = Y_{j,v}\theta_j + \eta_j, \tag{31}$$

where $v_j = [v_{j,p+1}\ v_{j,p+2}\ \ldots\ v_{j,T}]'$ is $T-p \times 1$, $\theta_j = [\theta_{j,1}\ \theta_{j,2}\ \ldots\ \theta_{j,p}]'$, is $p \times 1$ and $\eta_j = [\eta_{j,p+1}\ \eta_{j,p+2}\ \ldots\ \eta_{j,T}]'$ is $T-p \times 1$ and

$$Y_{j,v} = \begin{bmatrix} v_{j,p} & v_{j,p-1} & \cdots & v_{j,1} \\ v_{j,p+1} & v_{j,p} & \cdots & v_{j,2} \\ \vdots & \vdots & \vdots & \vdots \\ v_{j,T-1} & v_{j,T-2} & \cdots & v_{j,T-p} \end{bmatrix},$$

which is a $T-p \times p$ for $i = 1, 2, \ldots, N$.

Combining the priors described in section 3.3 with the likelihood function the following posterior distributions can be obtained:

The posterior of the AR-parameters of the idiosyncratic components is

$$\theta_j \sim N(\overline{\theta_j}, \overline{B}_{j,\theta}) I_{S_\theta}, \tag{32}$$

where

$$\overline{\theta_j} = \left(B_\theta^{-1} + (\sigma_{j,\eta}^2)^{-1} Y_{j,v}' Y_{j,v}\right)^{-1} \left((\sigma_{j,\eta}^2)^{-1} Y_{j,v}' v_j\right),$$

[36]See Chib [1993] and Kim and Nelson [1999a] for the estimation of β_j, $\theta_{j,1}, \theta_{j,2}, \ldots, \theta_{j,p}$, and σ_η and Zellner [1971] for the estimation of the VAR parameters.

and

$$\overline{B}_{j,\theta} = \left(B_\theta^{-1} + (\sigma_{j,\eta}^2)^{-1} Y_{j,v}' Y_{j,v}\right)^{-1}.$$

where I_{S_θ} is an indicator function enforcing stationarity.

The posterior of the variance of the idiosyncratic component $\sigma_{j,\eta}$ can be described as:

$$\sigma_{j,\eta} \sim \mathcal{IG}\left(\frac{(T+\alpha_\eta)}{2}, \frac{((v_j - Y_{j,v}\theta_i)'(v_j - Y_{j,v}\theta_i) + \delta_\eta)}{2}\right) \tag{33}$$

To estimate the factor loadings equation (3.1) can be rewritten as

$$x_j^* = \beta_j U^* + \eta, \tag{34}$$

where $x_j^* = [(1-\theta(L)_j)x_{j,p+1} \ (1-\theta(L)_i)x_{j,p+2} \dots \ (1-\theta(L)_i)x_{j,T}]'$ which is $T-p \times 1$, and $U^* = [(1-\theta(L)_j)U_{p+1} \ (1-\theta(L)_i)U_{p+2} \ \dots \ (1-\theta(L)_i)U_T]'$, which $T-p \times 1$ with $\theta(L)_j = (\theta_{j,1} + \theta_{j,2} + \dots + \theta_{j,p})$ for $j = 1, 2, \dots, N$. Thus, the posterior for the factor loadings is defined as:

$$\beta_j \sim N(\overline{\beta_j}, \overline{B}_{j,\beta}), \tag{35}$$

where

$$\overline{\beta_j} = \left(B_\beta^{-1} + (\sigma_{j,\eta}^2)^{-1} U^{*\prime} U^*\right)^{-1} \left((\sigma_{j,\eta}^2)^{-1} U^{*\prime} x_j^*\right)$$

and

$$\overline{B}_{i,\beta} = \left(B_\beta^{-1} + (\sigma_{j,\eta}^2)^{-1} U^{*\prime} U^*\right)^{-1}.$$

To estimate the VAR parameters equation (2.2) can rewritten as[37]

$$U = Y\Phi + \nu\,, \tag{36}$$

where $U \equiv [U_{q+1} \ U_{q+2} \ \dots \ U_T]'$ is a $T-q \times K$ matrix, $\Phi \equiv [\Phi_1 \ \Phi_2 \ \dots \ \Phi_q \ C]'$ is a $Kq+1 \times K$ matrix, and

$$Y \equiv \begin{bmatrix} Y_q' & Y_{q-1}' & \cdots & Y_1' & 1 \\ Y_{q+1}' & Y_q' & \cdots & Y_2' & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ Y_{T-1}' & Y_{T-2}' & \cdots & Y_{T-q}' & 1 \end{bmatrix}$$

[37]From now on, redefine $T \equiv T - p$.

is a $T-q \times Kq+1$ matrix including lagged U's. Thus the posterior of Φ is

$$\Phi|\Sigma_\nu \sim \mathcal{N}\left(vec(\overline{\Phi}), \overline{\Omega}\right) , \tag{37}$$

where $vec(\overline{\Phi}) \equiv \left(\underline{\Omega}^{-1} + \overline{\Omega}^{-1}\right)^{-1} \left(\underline{\Omega}^{-1} vec(\underline{\Phi}) + \overline{\Omega}^{-1} vec(\hat{\Phi})\right)$, with $\hat{\Phi} = (Y'Y)^{-1}Y'U$ and $\overline{\Omega} \equiv \Sigma_\nu \otimes (Y'Y)^{-1}$. The posterior for Σ_ν is inverted Wishart,

$$\Sigma_\nu \sim \mathcal{IW}(\hat{S} + \tau_2 \mathcal{I}_K, T+K+2),$$

where $\hat{S} \equiv (U - Y\hat{\Phi})'(U - Y\hat{\Phi})$ is the squared sample error matrix. Note that, as it is usually done in the literature, the VAR parameters are not restricted to be stationary.

3.2 Estimating the Stationary and Nonstationary Factors

To estimate the common latent factor, condition on the parameters of the model,[38] and rewrite the observation equation as

$$x^*_{j,t} = h_j \overline{U}_t + \eta_{j,t}, \tag{38}$$

where $\overline{U}_t = [U_t, U_{t-1}, \ldots, U_{t-q+1}]'$ is $Kq \times 1$, $x^*_{j,t} \equiv (1-\theta(L)_j)x_{j,t}$ with $\theta(L)_j \equiv \theta_{j,1} + \theta_{j,2} + \cdots + \theta_{j,p}$, and

$$H_j = [\beta_j \;\; -\theta_{j,1}\beta \;\; -\theta_{j,2}\beta_j \;\; \ldots \;\; \theta_{j,p}\beta_j \;\; 0_{1\times K(q-p-1)}],$$

which is a $1 \times Kq$ vector. Equation (38) can be rewritten as

$$X^*_t = HU_t + \eta_t, \tag{39}$$

where $X^*_t \equiv [x^*_{1,t} \; x^*_{2,t} \; \ldots \; x^*_{N,t}]'$, $\eta_t \equiv [\eta_{1,t} \; \eta_{2,t} \; \ldots \; \eta_{N,t}]'$ and

$$H \equiv \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_N \end{bmatrix}$$

Equation (39) can be rewritten as

$$\overline{U}_t = \Phi \overline{U}_{t-1} + \widetilde{\nu}_t, \tag{40}$$

[38]See Kim and Nelson [1999a]

where $\widetilde{\nu}_t = [\nu_t \; 0 \; \dots \; 0]'$ is $Kq \times 1$ and

$$\Phi = \left[\begin{array}{cccc} \Phi_1 & \Phi_2 & \cdots & \phi_q \\ & \mathcal{I}_{K(q-1)} & & 0_{K(q-1)\times K} \end{array} \right],$$

which is $Kq \times Kq$. For all empirical results shown below I use $q > p$.

To calculate the common factor, the algorithm suggested by Carter and Kohn [1994] and Frühwirth-Schnatter [1994] is used. This procedure draws the vector $\overline{U} = [\overline{U}_1 \, \overline{U}_2 \, \dots \, \overline{U}_T]$ from its joint distribution given by:

$$p(\overline{U}|\Psi, X) = p(\overline{U}_T|\Psi, X_T) \prod_{t=1}^{T-1} p(\overline{U}_t|\overline{U}_{t+1}, \Psi, X^t) \tag{41}$$

where $X^t = [X_1 \; X_2 \; \dots \; X_t]$. Because the error terms in equations (39) and (40) are Gaussian, equation (41)can be rewritten as

$$p(\overline{U}|\Psi, X) = \mathcal{N}(\overline{U}_{T|T}, P_{T|T}) \prod_{t=1}^{T-1} \mathcal{N}(\overline{U}_{t|t,\overline{U}_{t+1}}, P_{t|t,\overline{U}_{t+1}}), \tag{42}$$

with

$$\overline{U}_{T|T} = E(\overline{U}_T|\Psi, X), \tag{43}$$

$$P_{T|T} = Cov(\overline{U}_T|\Psi, X), \tag{44}$$

and

$$\overline{U}_{t|t,\overline{U}_{t+1}} = E(\overline{U}_t|\overline{U}_{t+1}, \Psi, X), \tag{45}$$

$$P_{t|t,\overline{U}_{t+1}} = Cov(\overline{U}_t|\overline{U}_{t+1}, \Psi, X), \tag{46}$$

The last step of the Kalman filter iteration results in $\overline{U}_{T|T}$ and $P_{T|T}$. They are used as the conditional mean and covariance matrix for the multivariate normal distribution $\mathcal{N}(\overline{U}_{T|T}, P_{T|T})$ to draw $\overline{U}_T$. To illustrate the Kalman Filter, start with the state-space system equations (39) and (40). The prediction step is

$$\overline{U}_{t|t-1} = \Phi\overline{U}_{t-1|t-1}, \tag{47}$$

$$P_{t|t-1} = \Phi P_{t-1|t-1}\Phi + Q, \tag{48}$$

where

$$Q = \begin{bmatrix} \Sigma_\nu & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix},$$

which is $Kq \times Kq$. To update these predictions the information from the forecast error

$$\kappa_t = X_t^* - H\overline{U}_{t|t-1} \tag{49}$$

is used, with variance

$$\Sigma = HP_{t|t-1}H' + \Sigma_\eta, \tag{50}$$

and the Kalman gain

$$K_t = P_{t|t-1}H'\Sigma^{-1}. \tag{51}$$

Thus, the updating equations are

$$\overline{U}_{t|t} = \overline{U}_{t|t-1} + K_t\kappa_t, \tag{52}$$

$$P_{t|t} = P_{t|t-1} + K_tHP_{t|t-1}. \tag{53}$$

To obtain draws for $\overline{U}_1, \overline{U}_2, \ldots, \overline{U}_{T-1}$, sample from $\mathcal{N}(\overline{U}_{t|t,\overline{U}_{t+1}}, P_{t|t,\overline{U}_{t+1}})$, using a backwards moving updating scheme, incorporating at time t information about $\overline{U}_t$ contained in period $t+1$:

$$\overline{U}_{t|t,\overline{U}_{t+1}} = \overline{U}_{t|t} + P_{t|t}\Phi' P_{t+1|t}^{-1}(\overline{U}_{t+1} - \overline{U}_{t+1|t}), \tag{54}$$

and

$$P_{t|t,\overline{U}_{t+1}} = P_{t|t} - P_{t|t}\Phi' P_{t+1|t}^{-1}\Phi P_{t|t}. \tag{55}$$

Appendix to Chapter 3

4 Estimation Procedure

4.1 Sampling from $\mathcal{P}(\Psi \mid z, \beta)$

To calculate the parameters summarized in Ψ we condition on values for z and β. However, for notational convenience we will not state this explicitly throughout the section.

VAR Parameters

We derive the posterior for the VAR parameters by using the prior specified in Section 4.5 and by combining them with the likelihood function described in this section. To make the description of the estimation procedure more convenient we rewrite Equation (4.2) as

$$Z = X\Phi + \epsilon^z \,, \tag{56}$$

where $Z \equiv [z_{p+1} \; z_{p+2} \; \dots \; z_T]'$ is a $T-p \times M$ matrix, $\Phi \equiv [\phi_1 \; \phi_2 \; \dots \; \phi_p \; c]'$ is a $Mp+1 \times M$ matrix, and

$$X \equiv \begin{bmatrix} z'_p & z'_{p-1} & \cdots & z'_1 & 1 \\ z'_{p+1} & z'_p & \cdots & z'_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ z'_{T-1} & z'_{T-2} & \cdots & z'_{T-p} & 1 \end{bmatrix}$$

is a $T-p \times Mp+1$ matrix including lagged Z's. Thus its likelihood function conditional on the first p observation can be expressed as

$$\mathcal{L}(\Phi, \Sigma_z) \propto |\Sigma_z|^{-\frac{T-p}{2}} \exp\left\{ tr\left\{ -\frac{1}{2}\Sigma_z^{-1}(Z - X\Phi)'(Z - X\Phi) \right\} \right\} \,, \tag{57}$$

where tr is the trace operator. The likelihood function can be decomposed into

$$\mathcal{L}(\Phi, \Sigma_z) \propto |\Sigma_z|^{-\frac{T-p}{2}} \exp\left\{ tr\left\{ -\frac{1}{2}\Sigma_z^{-1}\left(\hat{S} + \frac{1}{2}(\Phi - \hat{\Phi})'X'X(\Phi - \hat{\Phi}) \right) \right\} \right\} \,, \tag{58}$$

where $\hat{S} \equiv (Z - X\hat{\Phi})'(Z - X\hat{\Phi})$ is the SSE matrix, with $\hat{\Phi} \equiv (X'X)^{-1}X'Z$. Furthermore we subdivide it into the conditional density for Φ, taking values for Σ_z^{-1} as given,

$$\mathcal{F}(\Phi|\Sigma_z) \propto |\Sigma_z|^{-\frac{M}{2}} \exp\left\{-\frac{1}{2}\left(vec(\Phi) - vec(\hat{\Phi})\right)'\left(\Sigma_z^{-1} \otimes X'X\right)\left(vec(\Phi) - vec(\hat{\Phi})\right)\right\} \tag{59}$$

and the marginal density for Σ_z^{-1}

$$\mathcal{F}(\Sigma_z) \propto |\Sigma_z|^{-\frac{T-M-p}{2}} \exp\left\{tr\left\{-\frac{1}{2}\Sigma_z^{-1}\hat{S}\right\}\right\}. \tag{60}$$

Expression (59) is a normal density and Equation (60) a Wishart density. Thus the likelihood function can be described as a product of a normal density for Φ conditional on Σ_z and an inverted Wishart density for Σ_z,

$$\mathcal{L}(\Phi, \Sigma_z) \propto \mathcal{N}\left(vec(\hat{\Phi}), \Sigma_z \otimes X'X^{-1}\right)\mathcal{IW}\left(\hat{S}, TA - pM\right), \tag{61}$$

where for the inverted Wishart density $\hat{S}$ serves as the scale matrix and $TA - pM$ as the degrees of freedom. Combining the likelihood function with the conjugate prior described in Section 4.5, we obtain the following normal posterior for Φ,

$$\Phi|\Sigma_z \sim \mathcal{N}\left(vec(\overline{\Phi}), \Sigma_z \otimes \overline{X}'\overline{X}^{-1}\right), \tag{62}$$

where $\overline{\Phi} \equiv \overline{X}'\overline{X}^{-1}(X^{*\prime}Z^{*} + X'Z)$ with $\overline{X}'\overline{X} \equiv (X^{*\prime}X^{*} + X'X)$ and, as we assume an improper prior on Σ_z, the posterior is proportional to the second term described in Equation(61).

AR Parameters

As the error terms in equation (4.3) are independent of each other, we can estimate the AR parameters equation by equation. We rewrite Equation (4.3) as

$$\beta^i = G^i\alpha^i + \epsilon^{\beta^i} \quad \text{for} \quad i = 1, \ldots, M, \tag{63}$$

where $\beta^i \equiv [\beta^i_q\ \beta^i_{q+1}\ \ldots\ \beta^i_A]'$ is an $(A-q+1)\times 1$ vector, $\alpha^i \equiv [\alpha^i_1\ \alpha^i_2\ \ldots\ \alpha^i_q]'$ is a $q\times 1$ vector, $\epsilon^{\beta^i} \equiv [\epsilon^{\beta^i}_q\ {\epsilon^{\beta^i}}_{q+1}\ \ldots\ \epsilon^{\beta\, i}_A]'$, which is $(A-q+1)\times 1$ vector, and

$$G^i \equiv \begin{bmatrix} \beta^i_{q-1} & \beta^i_{q-2} & \cdots & \beta^i_0 \\ \beta^i_q & \beta^i_{q-1} & \cdots & \beta^i_1 \\ \vdots & \vdots & \ddots & \vdots \\ \beta^i_{A-1} & \beta^i_{A-2} & \cdots & \beta^i_{A-q} \end{bmatrix}$$

is an $(A-q+1)\times q$ matrix. Because we assume a flat prior for the AR parameters, the posterior of the AR parameters is proportional to the likelihood function. We can apply a similar decomposition as in Section 4.1 and obtain the following normal inverted gamma posterior

$$\mathcal{P}\left(\alpha^i, \sigma^i_\beta\right) = \mathcal{F}\left(\alpha^i|\sigma^i_\beta\right)\mathcal{F}\left(\sigma^i_\beta\right)\ . \tag{64}$$

The posterior for α^i conditional on the variance σ^i_β is

$$\alpha^i|\sigma^i_\beta \sim \mathcal{N}\left(\hat{\alpha}^i, \sigma^i_\beta(G^{i\prime}G^i)^{-1}\right)\ , \tag{65}$$

where $\hat{\alpha}^i$ is the ordinary least squares (OLS) estimate and the marginal posterior for σ^i_β is the inverted gamma distribution

$$\sigma^i_\beta \sim \mathcal{IG}\left(\frac{\hat{s}}{2}, \frac{(A-q)}{2}\right)\ , \tag{66}$$

where $\hat{s} = (\beta^i - G^i\alpha^i)'(\beta^i - G^i\alpha^i)$ is used as the scale parameter and $A-q$ as the degrees of freedom.

Variance

We assume the variances of the disturbances in Equation (4.1) to be the same for the dimensions $x = 0, 1, \ldots, A$ and $t = 1, 2, \ldots, T$. Hence the posterior can be expressed as the inverted gamma distribution

$$\sigma^2_d \sim \mathcal{IG}\left(\frac{TA+\tau_1}{2}, \frac{\hat{s}_d+\tau_2}{2}\right)\ , \tag{67}$$

where $\hat{s}_d = \sum_{t=1}^T \sum_{x=0}^A \left(d_{x,t} - \overline{d}_x - \beta_x z_t\right)^2$.

4.2 Sampling from $\mathcal{P}(z \mid \Psi, \beta)$

To calculate the latent z we condition on values for Ψ and β. However, for notational convenience we will not state this explicitly throughout the section. As z contains latent variables, we set up a state space system, which we will describe in the following.

We rewrite Equation (4.2) into its canonical form and use it as our state equation

$$Z_t = \widetilde{\Phi} Z_{t-1} + \widetilde{\epsilon}_t^z\,, \tag{68}$$

where $Z_t \equiv [z_t\ z_{t-1}\ \ldots\ z_{t-p+1}1]'$ is $(Mp+1) \times 1$, which is the state vector, $\widetilde{\epsilon}_t^z \equiv [\epsilon_t^z\ 0\ \ldots\ 0]'$, which is a an $(Mp+1) \times 1$ vector, and

$$\widetilde{\Phi} \equiv \begin{bmatrix} \phi_1 & \ldots & \phi_p & c \\ & I_{M(p-1)\times M(p-1)} & & 0_{M(p-1)\times(M+1)} \\ 0 & \ldots & 0 & 1 \end{bmatrix},$$

which is an $(Mp+1) \times (Mp+1)$ matrix, where I is the identity matrix.

To derive our observation equation we first rewrite Equation (4.1) as

$$\overline{D}_t = \ \beta z_t + \epsilon_t^d\,, \tag{69}$$

with

$$\overline{D}_t \equiv \begin{bmatrix} D_t \ - \overline{D} \\ Y_t \end{bmatrix},$$

which is an $(A+N) \times 1$ matrix, with $D_t \equiv [d_{0,t}\ d_{1,t}\ \ldots d_{A,t}]'$, $\overline{D} \equiv [\overline{d}_0\ \overline{d}_1\ \ldots \overline{d}_A]'$, where both are $A \times 1$ vectors, $\epsilon_t^d = [\epsilon_{0,t}^d\ \epsilon_{1,t}^d\ \ldots\ \epsilon_{A,t}^d\ 0_{1\times N}]'$ is a $(A+N)\times 1$, and

$$\beta \equiv \begin{bmatrix} \beta^\kappa & \beta^Y \\ 0_{N\times K} & I_{N\times N} \end{bmatrix},$$

which is an $(A+N) \times M$ matrix, with $\beta^\kappa \equiv [(\beta_0^\kappa)'\ (\beta_1^\kappa)' \ldots\ (\beta_A^\kappa)']'$, which is an $A \times K$ matrix, and $\beta^Y \equiv \left[(\beta_0^Y)'\ (\beta_1^Y)' \ldots\ (\beta_A^Y)'\right]'$, which is an $A \times N$ matrix.

We rewrite Equation (69) to match the state equation and finally obtain our observation equation

$$\overline{D}_t = \ H Z_t + \epsilon_t^d\,, \tag{70}$$

where $H \equiv [\beta \; 0_{A+N \times M(p-1)+1}]$ is an $(A+N) \times (Mp+1)$ matrix.

The latent variable z can be estimated using the method desribed in Carter and Kohn [1994] and Frühwirth-Schnatter [1994].[39] We draw z from its joint distribution

$$\mathcal{P}(z|D) = \mathcal{P}\left(z_T|\overline{D}_T\right) \prod_{t=1}^{T-1} \mathcal{P}\left(z_t|z_{t+1}, D^t\right) \,, \tag{71}$$

where $D = [\overline{D}_1 \; \overline{D}_2 \; \ldots \; \overline{D}_T]$ and $D^t = [\overline{D}_1 \; \overline{D}_2 \; \ldots \; \overline{D}_t]$. Because the disturbances in Equations (68) and (70) are Gaussian, Equation (71) can be rewritten as

$$\mathcal{P}(z|D) = \mathcal{N}(z_{T|T}, P_{T|T}) \prod_{t=1}^{T-1} \mathcal{N}(z_{t|t,z_{t+1}}, P_{t|t,z_{t+1}}) \,, \tag{72}$$

with

$$z_{T|T} = E(z_T|D) \,, \tag{73}$$

$$P_{T|T} = Cov(z_T|D) \,, \tag{74}$$

and

$$z_{t|t,z_{t+1}} = E(z_t|z_{t+1}, D) \,, \tag{75}$$

$$P_{t|t,z_{t+1}} = Cov(z_t|z_{t+1}, D) \,. \tag{76}$$

We obtain $z_{T|T}$ and $P_{T|T}$ from the last step of the Kalman filter iteration and use them as the conditional mean and covariance matrix for the multivariate normal distribution $\mathcal{N}(z_{T|T}, P_{T|T})$ in order to draw z_T. In the following we will describe the Kalman filter procedure.

We begin with the prediction steps

$$z_{t|t-1} = \widetilde{\Phi} z_{t-1|t-1} \,, \tag{77}$$

$$P_{t|t-1} = \widetilde{\Phi} P_{t-1|t-1} \widetilde{\Phi} + Q \,, \tag{78}$$

where

$$Q \equiv \begin{bmatrix} \Sigma_z & 0_{M \times M(p-1)+1} \\ 0_{M(p-1)+1 \times M(p-1)+1} & 0_{M(p-1)+1 \times M} \end{bmatrix} ,$$

[39] See also Kim and Nelson [1999a].

which is an $(Mp+1) \times (Mp+1)$ matrix. Accordingly, the forecast error is

$$\nu_t = \overline{D}_t - H z_{t|t-1}\,, \tag{79}$$

with the corresponding variance

$$\Omega = H P_{t|t-1} H' + R\,, \tag{80}$$

where $R \equiv \sigma_d^2 I_N$. The Kalman gain can be expressed as

$$K_t = P_{t|t-1} H' \Omega^{-1}\,. \tag{81}$$

Thus the updating equations are

$$z_{t|t} = z_{t|t-1} + K_t \nu_t\,, \tag{82}$$

$$P_{t|t} = P_{t|t-1} + K_t H P_{t|t-1}\,. \tag{83}$$

To obtain draws for $z_1, z_2, \ldots, z_{T-1}$ we sample from $N\left(z_{t|t,z_{t+1}}, P_{t|t,z_{t+1}}\right)$, using a backward moving updating scheme, incorporating at time t information about z_t contained in period $t+1$. More precisely, we move backward and generate z_t for $t = T-1, \ldots, 1$ at each step while using information from the Kalman filter and z_{t+1} from the previous step. The updating equations are

$$z_{t|t,z_{t+1}} = z_{t|t} + P_{t|t} \Phi' P_{t+1|t}^{-1} (z_{t+1} - z_{t+1|t}) \tag{84}$$

and

$$P_{t|t,F_{t+1}} = P_{t|t} - P_{t|t} \Phi' P_{t+1|t}^{-1} \Phi P_{t|t}\,. \tag{85}$$

4.3 Sampling from $\mathcal{P}(\beta \mid \Psi, z)$

To calculate β we take values for Ψ and z as given. The procedure applied here is very similar to the one described in Section 4.2. Hence we will just give a brief overview of the estimation procedure. However, there is one important difference, namely, that now we move in the age dimension $x = 0, 1, \ldots, A$ and not in $t = 1, 2, \ldots, T$ as in Section 4.2.

Our state equation can be expressed as

$$\widetilde{\beta}_x = \widetilde{\alpha}\widetilde{\beta}_{x-1} + \widetilde{\epsilon}_x^{\beta}\,, \tag{86}$$

where $\widetilde{\beta}_x = [\beta_{x-1}\ \beta_{x-2}\ \ldots\ \beta_{x-q+1}]'$ is $Mq \times 1$, which is denoted as the state vector, $\widetilde{\epsilon}_x^{\beta} = [\epsilon_x^{\beta}\ 0\ \ldots\ 0]'$ is $Mq \times 1$, and

$$\widetilde{\alpha} = \begin{bmatrix} \alpha_1 & \ldots & \alpha_q \\ I_{M(p-1)\times M(p-1)} & & 0_{M(p-1)\times(M+1)} \end{bmatrix},$$

which is an $Mq \times Mq$ matrix. Hence our observation equation can be expressed as

$$\widetilde{D}_x - \overline{d}_x = W\widetilde{\beta}_x + \epsilon_x^d\,, \tag{87}$$

where $\widetilde{D}_x \equiv [d_{x,1}\ d_{x,2}\ \ldots, d_{x,T}]'$ is a $T\times 1$ vector, $\epsilon_x^d \equiv [\epsilon_{x,1}^d\ \epsilon_{x,1}^d \ldots\ \epsilon_{x,1}^d]$ is a $T \times 1$ vector, and $W \equiv [z'\ 0_{T,M(q-1)}]$ is a $T \times Mq$ matrix. For $x = 0, 1, \ldots, A$ instead of $t = 1, 2, \ldots T$, $\widetilde{\Phi} \equiv \widetilde{\alpha}$, $H \equiv W$, $R \equiv \sigma_d^2 I_T$, and

$$Q \equiv \begin{bmatrix} \Sigma_\beta & 0_{M\times M(q-1)} \\ 0_{M(q-1)\times M(q-1)} & 0_{M(q-1)\times M} \end{bmatrix},$$

we can apply the procedure described in Section 4.2 to calculate β.

5 Life Table Calculations

We use both observed and estimated age-specific death rates $m_{x,t}$ to calculate period life tables by single years of age and time and present the results for the probability $l_{x,t}$ of surviving up to the exact age x and the probability $d_{x,t}$ of dying at age x. Both variables represent birth time probabilities for all born living. Thus they are unconditional. In contrast to this, the remaining life expectancy $e_{x,t}$ is conditional on still being alive at exact age x. The respective calculations are standard.[40]

The conditional probability of dying before arriving at exact age $x + 1$ if still alive

[40] See Preston et al. [2005, pp. 38–54] or Wilmoth et al. [2007, pp. 35–39]. Unlike the life table calculations of the Human Mortality Database, we do not smooth observed death rates $m_{x,t}$ for the higher age classes at the beginning of the calculations.

at exact age x is

$$q_{x,t} \equiv \frac{m_{x,t}}{1 + (1 - \alpha_{x,t})m_{x,t}} \ .$$

The factor $\alpha_{x,t}$ reflects the average fraction of a year that people dying at age x still live after their xth birthday. For infants, with their high mortality in the first weeks, we apply, according to Preston et al. [2005, pp. 47–48] and Wilmoth et al. [2007, p. 38], sex-specific values originally proposed by Coale and Demeny [1983]:

$$\alpha_{0,t}^{male} \equiv \begin{cases} 0.045 + 2.684 m_{0,t}^{male} & , m_{0,t}^{male} < 0.107 \\ 0.330 & , m_{0,t}^{male} \geq 0.107 \end{cases}$$

and

$$\alpha_{0,t}^{female} \equiv \begin{cases} 0.053 + 2.800 m_{0,t}^{female} & , m_{0,t}^{female} < 0.107 \\ 0.350 & , m_{0,t}^{female} \geq 0.107 \end{cases}$$

Consistent values for $\alpha_{0,t}^{total}$ would require information about the total numbers of deaths for both sexes to weight the respective values for $m_{0,t}^{male}$ and $m_{0,t}^{female}$. Instead of that, when using the total figures of both sexes combined, we adopt a simple approximation roughly reflecting the higher infant mortality and higher birth rates of males

$$\alpha_{0,t}^{total} \equiv 0.56\alpha_{0,t}^{male} + 0.44\alpha_{0,t}^{female} \ ,$$

which does not perceivably influence the results. The highest recorded age class $\tilde{x}$ is open, that is, not restricted to 1 year. We set $\alpha_{\tilde{x},t} \equiv \frac{1}{m_{\tilde{x},t}}$ resulting in $q_{\tilde{x},t} = 1$. For all other age classes $0 < x < \tilde{x}$ we assume a uniform distribution of cases of death and apply

$$\alpha_{x,t} \equiv 0.5 \ .$$

The conditional probability of surviving up to exact age $x+1$ if still alive at exact age x is

$$p_{x,t} \equiv 1 - q_{x,t} \ .$$

For all born living the unconditional probability of surviving up to exact age x is

$$l_{x,t} \equiv l_{0,t} \prod_{i=0}^{x-1} p_{i,t} = l_{x-1,t}p_{x-1,t}$$

and the unconditional probability of dying at age x is

$$d_{x,t} \equiv l_{0,t} \prod_{i=0}^{x-1} p_{i,t} q_{x,t} = l_{x,t} q_{x,t} \; .$$

We normalize $l_{0,t} \equiv 1$ to get values for $l_{x,t}$ and $d_{x,t}$ interpretable as probabilities for the life table population. The alternative choice of $l_{0,t} \equiv 100000$ would result in the numbers $l_{x,t}$ and $d_{x,t}$ of survivors and deaths out of $100,000$ live births.

The person-years lived at age x and from age x onward are

$$L_{x,t} \equiv l_{x,t} - (1 - \alpha_{x,t}) d_{x,t}$$

and

$$T_{x,t} \equiv \sum_{i=x}^{\tilde{x}} L_{i,t} \; .$$

Finally, we get the conditional remaining life expectancy if still alive at exact age x

$$e_{x,t} \equiv \frac{T_{x,t}}{l_{x,t}} \; .$$

Note that all variables in a period life table refer to the same point in time t and reflect its time-specific conditions. Variables such as $l_{x,t}$, $d_{x,t}$, and $e_{x,t}$ that are aggregated from the basic variables of several age classes are synthetic measures for this period. They mix up the values of the different age classes belonging to different cohorts because they correspond to a cross section of the Lexis diagram. Hence the aggregated variables of a period life table do not describe the conditions for the members of any real age cohort, who pass through many different periods but are always subject to the mortality of their very own cohort. To analyze these conditions along the life cycle, cohort life tables, which are calculated from data of a single cohort, are adequate and correspond to diagonal sections of the Lexis diagram. Unfortunately, they can only be accurately calculated retrospectively. Of course, short-run fluctuations that last only a few periods but affect many age classes have a greater effect on period life tables than on cohort life tables. The latter exhibit, in general, less volatility, because time-specific anomalies are not wrongly extrapolated but on the contrary often counterbalanced later on.

Appendix to Chapter 4

6 Additional Figures

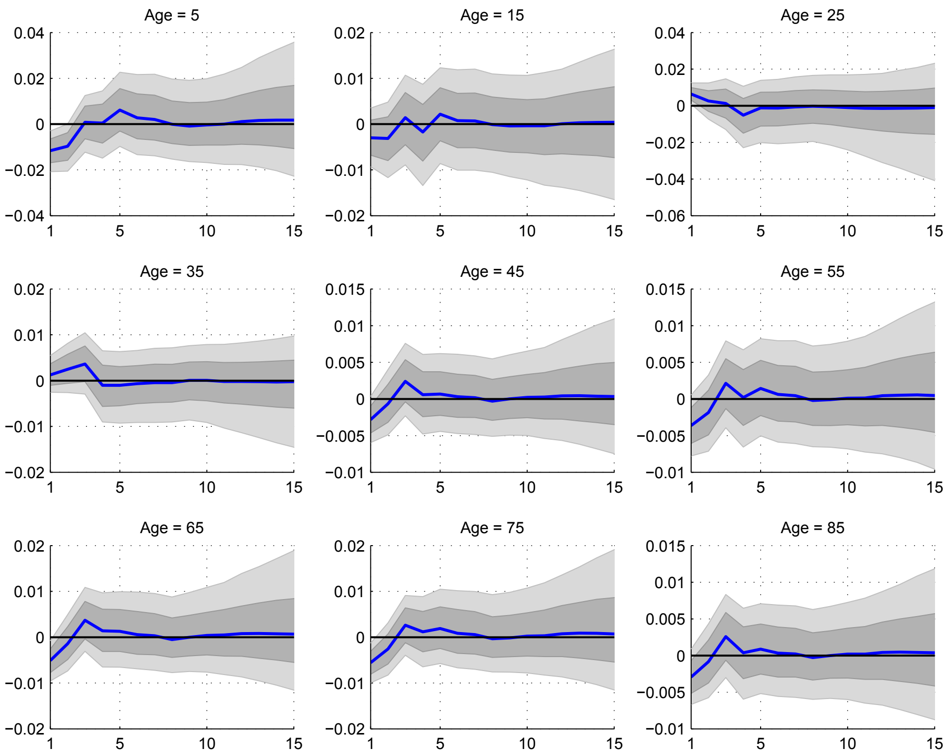

Figure 10: Impulse responses of log mortality at some fixed ages of males in France to a one standard deviation shock in unemployment occurring in year 1. The entire gray shaded area around the blue median represents 90% of the posterior probability mass and the dark gray shaded area represents 68% of the posterior probability mass.

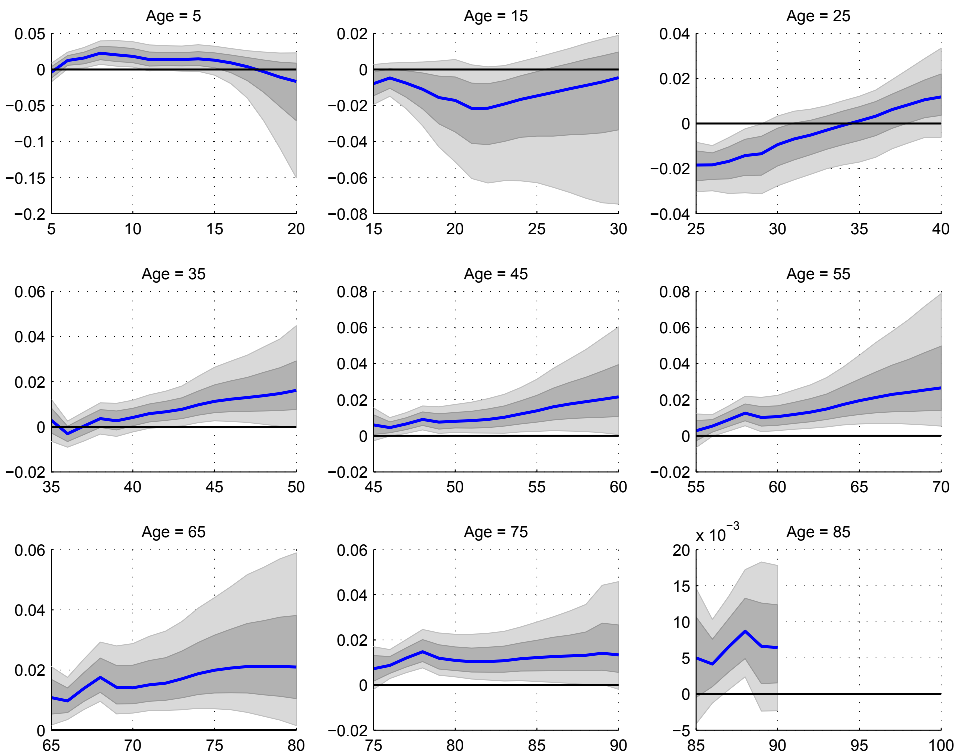

Figure 11: Impulse responses of log mortality in the further life of some male cohorts in France to a one standard deviation shock in GDP growth occurring at the labeled age. The entire gray shaded area around the blue median represents 90% of the posterior probability mass and the dark gray shaded area represents 68% of the posterior probability mass.

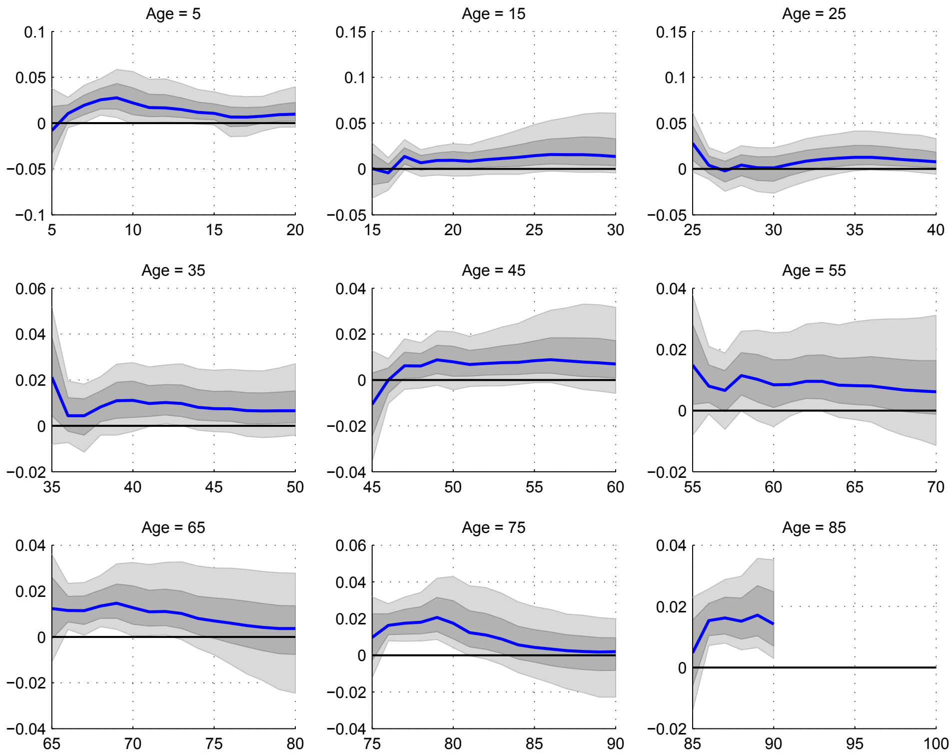

Figure 12: Impulse responses of log mortality in the further life of some male cohorts in Japan to a one standard deviation shock in GDP growth occurring at the labeled age. The entire gray shaded area around the blue median represents 90% of the posterior probability mass and the dark gray shaded area represents 68% of the posterior probability mass.

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Selbständigkeitserklärung

Ich erkläre, dass ich die vorliegende Arbeit selbständig und nur unter Verwendung der angegebenen Literatur und Hilfsmittel angefertigt habe.

Berlin, den 08. Juni 2009 Samad Sarferaz